Fairness in Participatory Budgeting via Equality of Resources

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Joint work with Jan Maly, Ulle Endriss and Martin Lackner

Institute for Logic, Language and Computation (ILLC)
University of Amsterdam

2022 SSCW Meeting

1. Introduction

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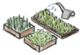

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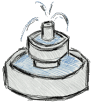

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Participatory Budgeting

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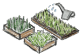

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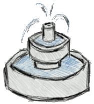

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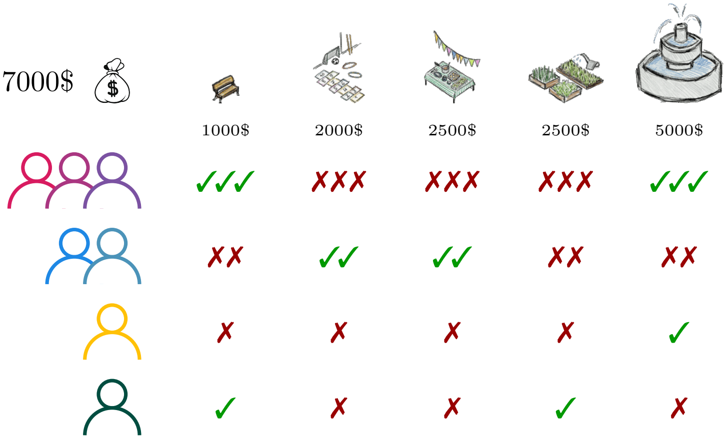
Standard Model of Participatory Budgeting



Fairness Requirements

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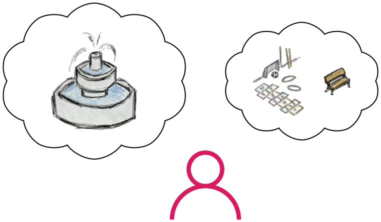
Incentive Compatibility



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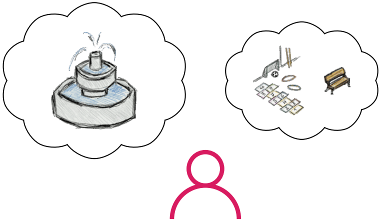
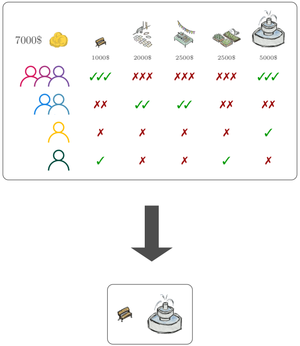


Algorithmic Perspective



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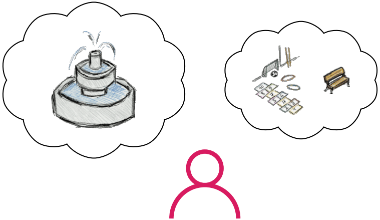

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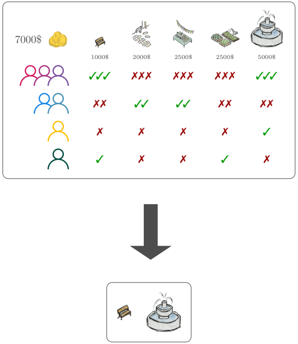

Satisfaction-Based Fairness for Participatory Budgeting

Fairness is about distributing some *measure* fairly among the agents.

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We aim at *equity of resources* among the agents.

2. The Share

Definition

The share of an agent:
the ressources spent on
that specific agent

$$share(\pi, A_i) = \sum_{p \in \pi \cap A_i} \frac{c(p)}{|\{A' \in \boldsymbol{A} \mid p \in A'\}|}$$

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The budget allocation

The agent's ballot

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Cost of the project

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The budget allocation

The agent's ballot

Number of voters approving of p

An Example

= 10	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	
	✓		✓			
	✓			✓		

An Example

= 10	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	4
	✓		✓			
	✓			✓		

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	✓	✓		✓	✓	4
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3. Providing Fair Share

The Perfect Situation

Every agent is provided their *fair share*, *i.e.*:

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Cost	6	2	2	4	5	**(Fair) Share**
Agent 1	✓	✓		✓	✓	
Agent 2	✓		✓			
Agent 3	✓			✓		

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Budget $= 10$

	1	2	3	4	5	
Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	4; 3 + 1/3
	✓		✓			2; 3 + 1/3
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A First Problem

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Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	7 / $3 + 1/3$
	✓		✓			2 / $3 + 1/3$
	✓			✓		0 / $3 + 1/3$

= 10

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	✓			✓		2 / 3 + 1/3

It is not possible to always provide fair share to everyone (and hard to know if we can).

4. Experimental Analysis of the Share

Experimental Analysis

Instances: 353 instances from Pabulib with up to 65 projects.

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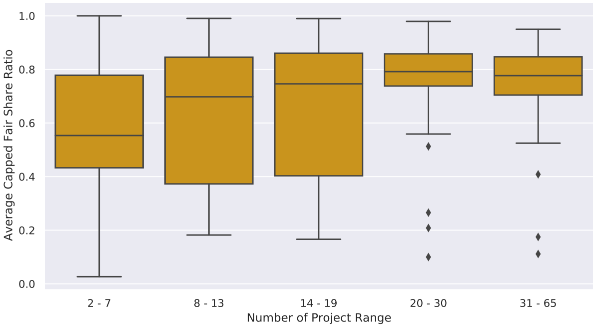
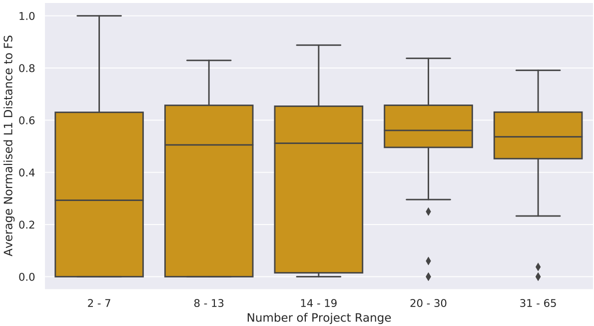
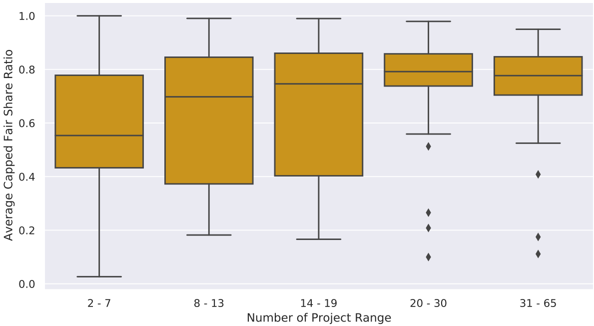
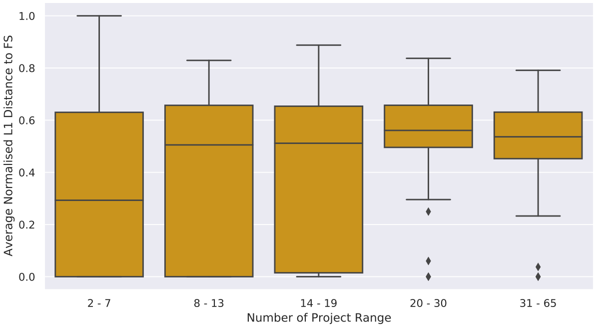
Fair share can be provided in only one instance out of the 353 considered (with 3 projects and 198 voters).

Optimising the Measures of Interest









➥ We are far from achieving fair share.

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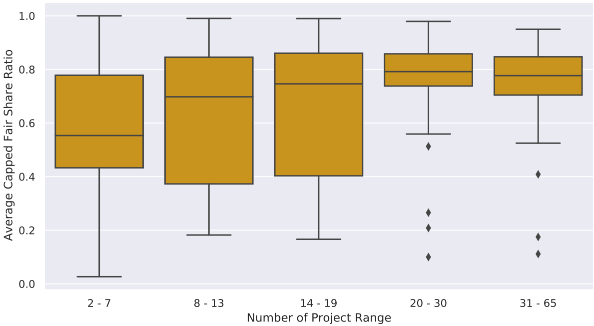
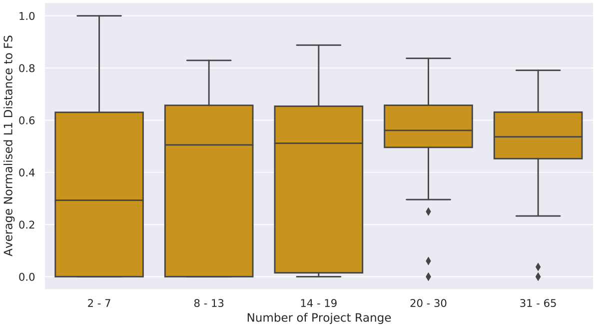
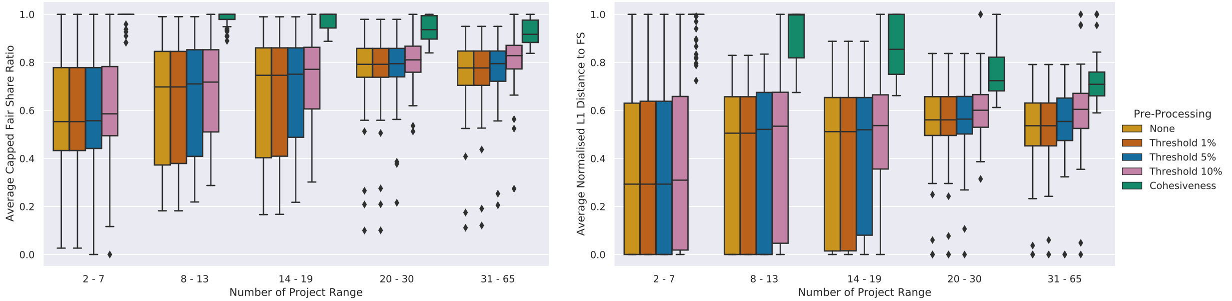




- We are far from achieving fair share.
- It gets easier as the number of projects increase.

Optimal Average Fair Share Ratio – Preprocessing



↳ Fair Share is hard to satisfy, *structurally* hard.

5. Approximate Fair Share

Two Relaxations — Fair Share up to One Project

Every agent is provided their *fair share up to one project*, *i.e.*, for each agent there exists a project $p \in \mathcal{P}$ such that:

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↪ This is however still unsatisfiable (and hard again)...

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A budget allocation π provides *local fair share* if there is no project $p \in \mathcal{P} \setminus \pi$ such that for every agent i approving of p we have:

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↳ An explanation? If such a p exists, all supporters of p receive less than their fair share and:

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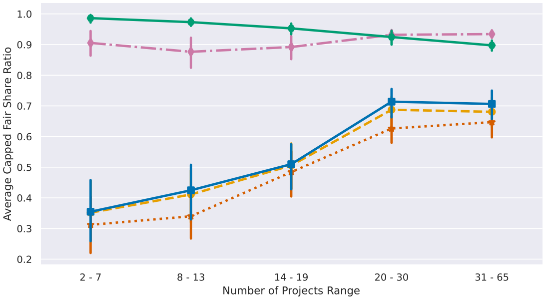
↳ But how does MES performs in terms of fair share?

6. Achieved Fair Share by Common Rules

Distance to Fair Share



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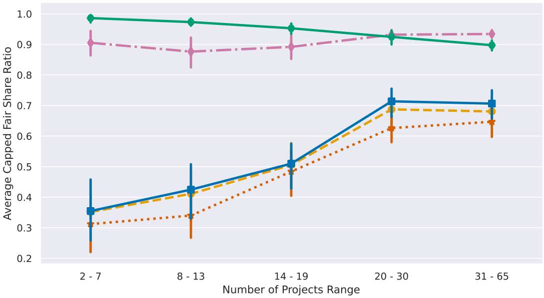

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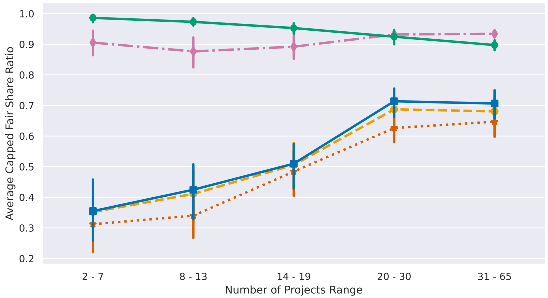

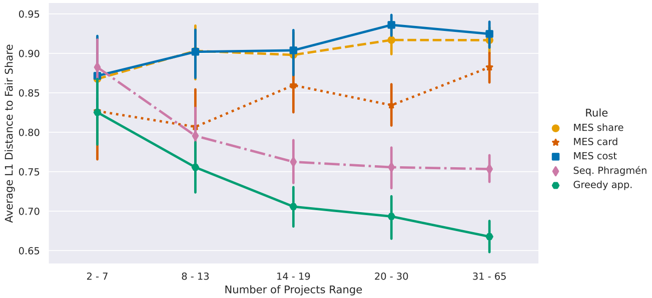

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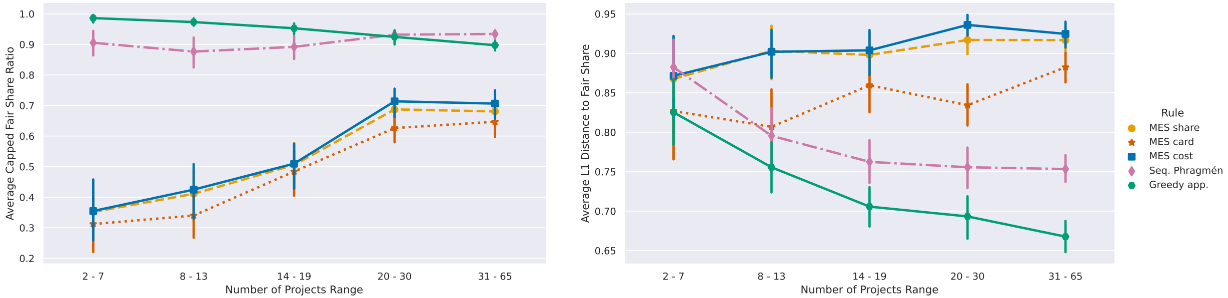

- The capped fair share ratio is not a good measure because it is correlated to *the budget used*.
- MES rules approach fair share nicely, and MES_{cost} is particularly attractive.

7. Conclusion

Wrap-Up

We have...

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- ...Presented the share, one operationalisation of this idea;
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Thanks!