Effort-Based Fairness for Participatory Budgeting

Simon Rey

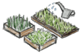
Joint work with Jan Maly, Ulle Endriss and Martin Lackner

1. Introduction

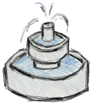
1000€

2000€

2500€



2500€



5000€

 : 7000€

Participatory Budgeting

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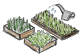
1000€

2000€

2500€



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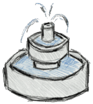


5000€

: 7000€

7000€

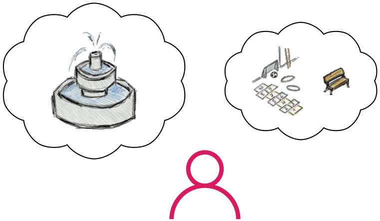
	1000€	2000€	2500€	2500€	5000€
(3 voters)	✓✓✓	✗✗✗	✗✗✗	✗✗✗	✓✓✓
(2 voters)	✗✗	✓✓	✓✓	✗✗	✗✗
(1 voter)	✗	✗	✗	✗	✓
(1 voter)	✓	✗	✗	✓	✗

Participatory Budgeting in the ComSoC Literature

Fairness Requirements

Fairness Requirements

Incentive Compatibility



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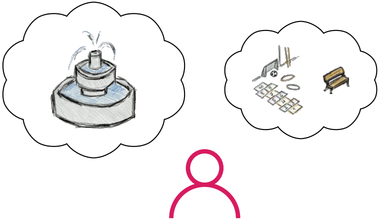
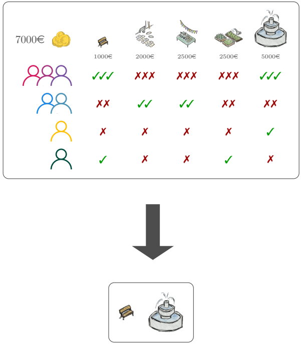


Algorithmic Perspective



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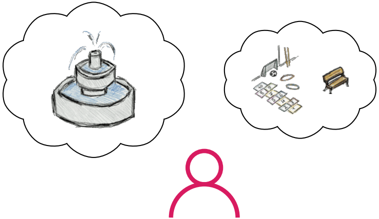

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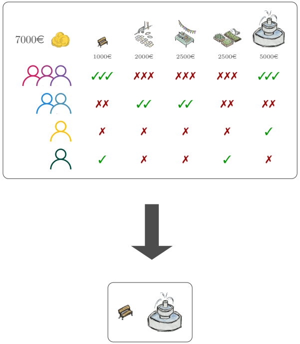

Fairness for Participatory Budgeting

Fairness is about distributing some *measure* fairly among the agents.

↳ What is a good measure in the case of participatory budgeting?

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Usually we consider distributing *satisfaction* equally among the agents but...

- ✗ Knowing the satisfaction of an agent is not an easy task
- ✗ Asking for utility functions is impossible in practice
- ✗ It is unclear how to derive satisfaction functions from approval ballots

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Cardinality Satisfaction	Cost Satisfaction
$\|A \cap \pi\|$	$c(A \cap \pi)$

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Cardinality Satisfaction	Cost Satisfaction
$\lvert A \cap \pi \rvert$	$c(A \cap \pi)$

We focus on distributing the *effort* spent on the agents fairly.

2. The Share

Definition

The share of an agent:
the effort spent on that
specific agent

$$share(\pi, A_i) = \sum_{p \in \pi \cap A_i} \frac{c(p)}{|\{A' \in \boldsymbol{A} \mid p \in A'\}|}$$

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The budget allocation

The agent's ballot

Definition

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Cost of the project

$$share(\pi, A_i) = \sum_{p \in \pi \cap A_i} \frac{c(p)}{|\{A' \in \boldsymbol{A} \mid p \in A'\}|}$$

The budget allocation

The agent's ballot

Number of voters approving of p

An Example

= 10	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	
	✓		✓			
	✓			✓		

An Example

= 10

	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	4
	✓		✓			
	✓			✓		

An Example

= 10

	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	4
	✓		✓			2
	✓			✓		

An Example

= 10	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	4
	✓		✓			2
	✓			✓		4

3. Providing Fair Share

The Perfect Situation

Every agent is provided their *fair share*, *i.e.*:

$$share(\pi, A_i) \geq \min\left\{share(A_i, i), \frac{b}{n}\right\}$$

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	✓		✓			
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Budget $= 10$

Cost	6	2	2	4	5	(Fair) Share
Agent 1	✓	✓		✓	✓	4; 3 + 1/3
Agent 2	✓		✓			2; 3 + 1/3
Agent 3	✓			✓		

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Budget $= 10$

	Project 1	Project 2	Project 3	Project 4	Project 5	
Cost	6	2	2	4	5	**(Fair) Share**
Agent 1	✓	✓		✓	✓	4 / $3 + 1/3$
Agent 2	✓		✓			2 / $3 + 1/3$
Agent 3	✓			✓		4 / $3 + 1/3$

A First Problem

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	4; 3 + 1/3
	✓		✓			4; 3 + 1/3
	✓			✓		2; 3 + 1/3

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	4; 3 + 1/3
	✓		✓			2; 3 + 1/3
	✓			✓		4; 3 + 1/3

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	4; 3 + 1/3
	✓		✓			2; 3 + 1/3
	✓			✓		2; 3 + 1/3

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	7; 3 + 1/3
	✓		✓			2; 3 + 1/3
	✓			✓		0; 3 + 1/3

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	7; 3 + 1/3
	✓		✓			0; 3 + 1/3
	✓			✓		2; 3 + 1/3

A First Problem

$\$ = 10$	1	2	3	4	5	
Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	4; 3 + 1/3
	✓		✓			4; 3 + 1/3
	✓			✓		2; 3 + 1/3

$\$ = 10$	1	2	3	4	5	
Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	4; 3 + 1/3
	✓		✓			2; 3 + 1/3
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Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	4; 3 + 1/3
	✓		✓			2; 3 + 1/3
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$\$ = 10$	1	2	3	4	5	
Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	7; 3 + 1/3
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Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	7; 3 + 1/3
	✓		✓			0; 3 + 1/3
	✓			✓		2; 3 + 1/3

It is not possible to always provide fair share to everyone.

A Second (Unsurprising) Problem

For a given instance, checking whether there is a budget allocation providing fair share is a strongly NP-complete problem (even with unit-cost).

The reduction is based on 3-Set-Cover.

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The reduction is based on 3-Set-Cover.

↳ What should we do then? Study *approximation* of the fair share.

4. Approximate Fair Share

Two Relaxations — Fair Share up to One Project

Every agent is provided their *fair share up to one project*, *i.e.*, for each agent there exists a project $p \in \mathcal{P}$ such that:

$$share(\pi \cup \{p\}, A_i) \geq \min\left\{share(A_i, i), \frac{b}{n}\right\}$$

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↳ This is however still unsatisfiable...

$= 5$	1	2	3
Cost	3	3	3
	✓	✓	
	✓		✓
		✓	✓

Two Relaxations — Local Fair Share

A budget allocation π provides *local fair share* if there is no project $p \in \mathcal{P} \setminus \pi$ such that for every agent i approving of p we have:

$$share(\pi \cup \{p\}, A_i) < \min\left\{share(A_i, i), \frac{b}{n}\right\}$$

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↳ An explanation? If such a p exists, all supporters of p receive less than their fair share and:

- Either p can be selected without exceeding the budget limit; let's select it then!
- Or, some voter $i^\star$ received more than their fair share; let's then exchange a project approved by $i^\star$ with p!

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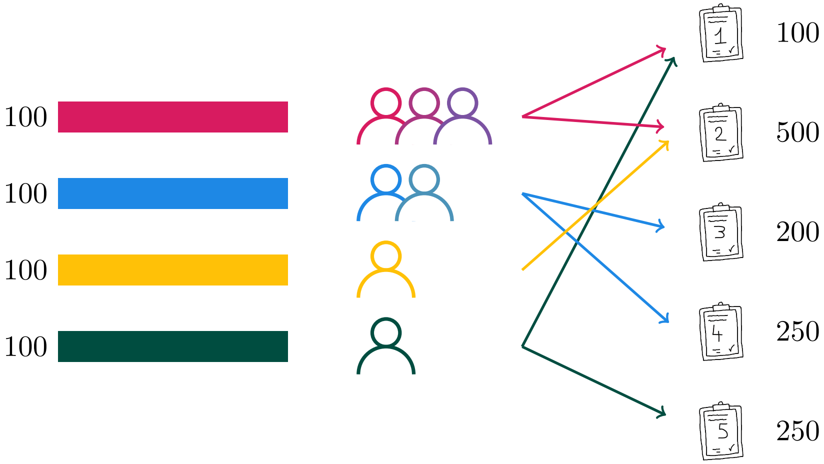
Note: This concepts is provably independent from fair share up to one project, *i.e.*, some budget allocations satisfy one but not the other, and vice versa.

Finally Some Good News

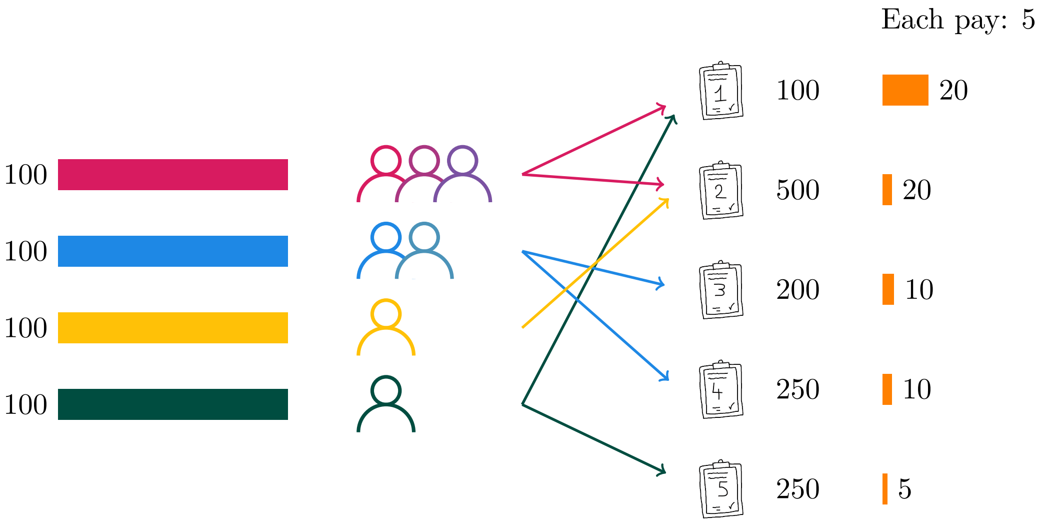
Local fair share is satisfiable in polynomial time!!!

➥ We can prove that *Rule X* (a.k.a. the method of equal share) satisfies local fair share.

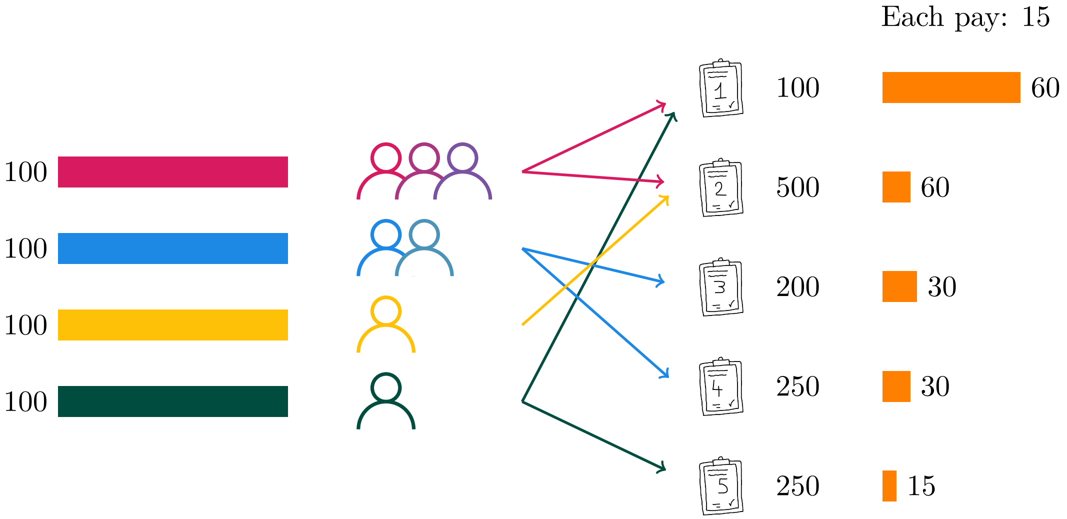
Rule X



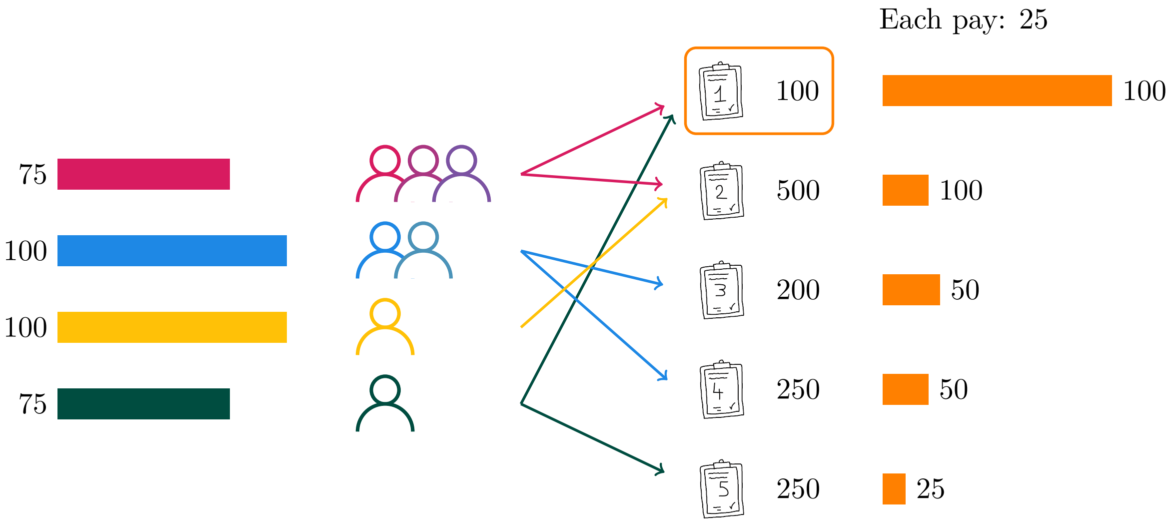
Rule X



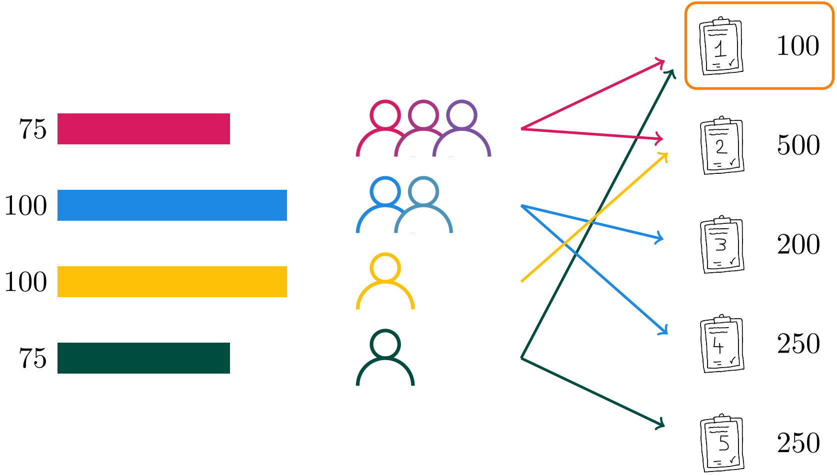
Rule X



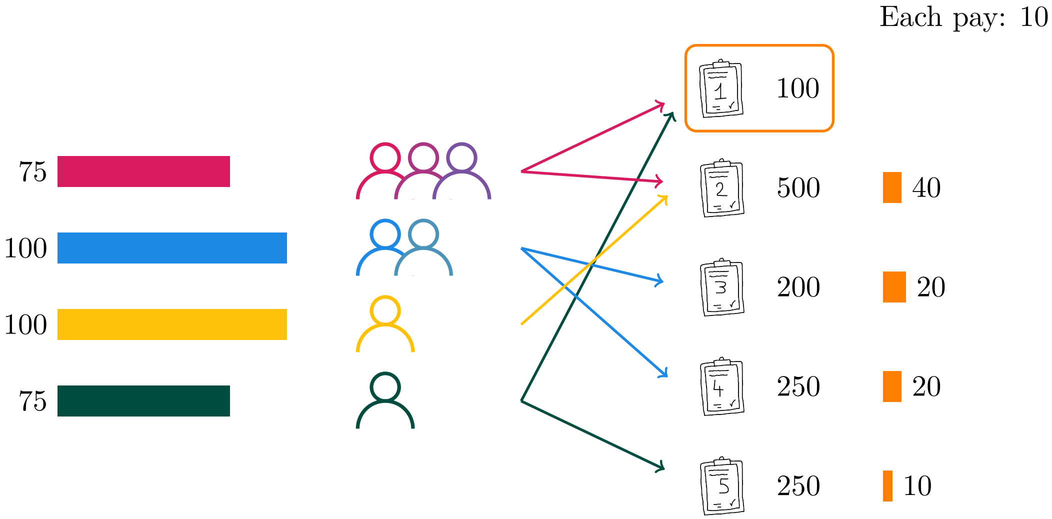
Rule X


Each pay: 25
75
100
100
75
1
2
3
4
5
100
500
200
250
250
100
100
50
50
25

Rule X

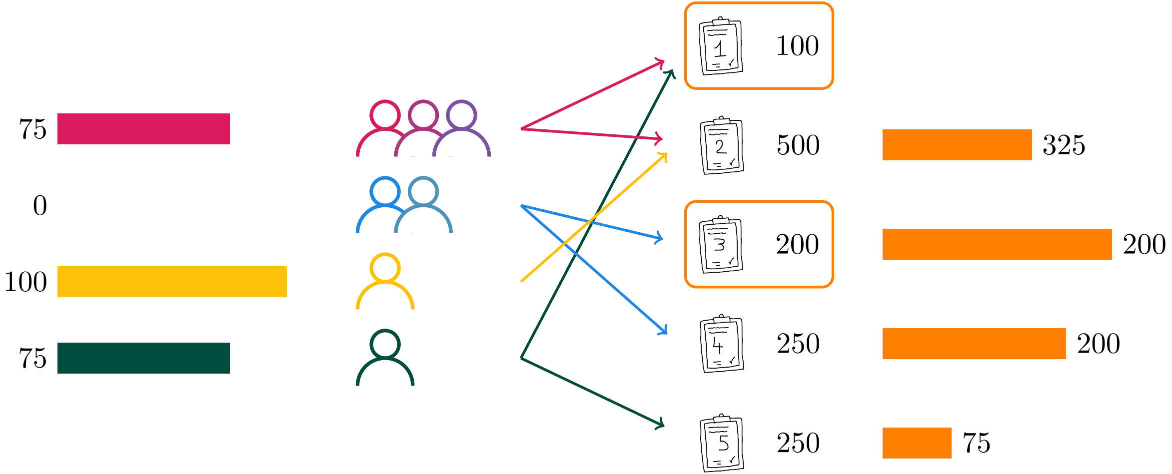


Rule X

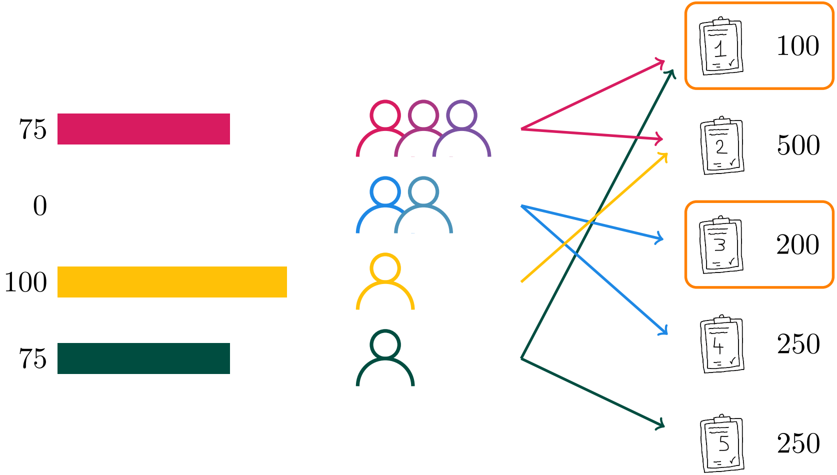


Rule X

Each pay at most: 100



Rule X



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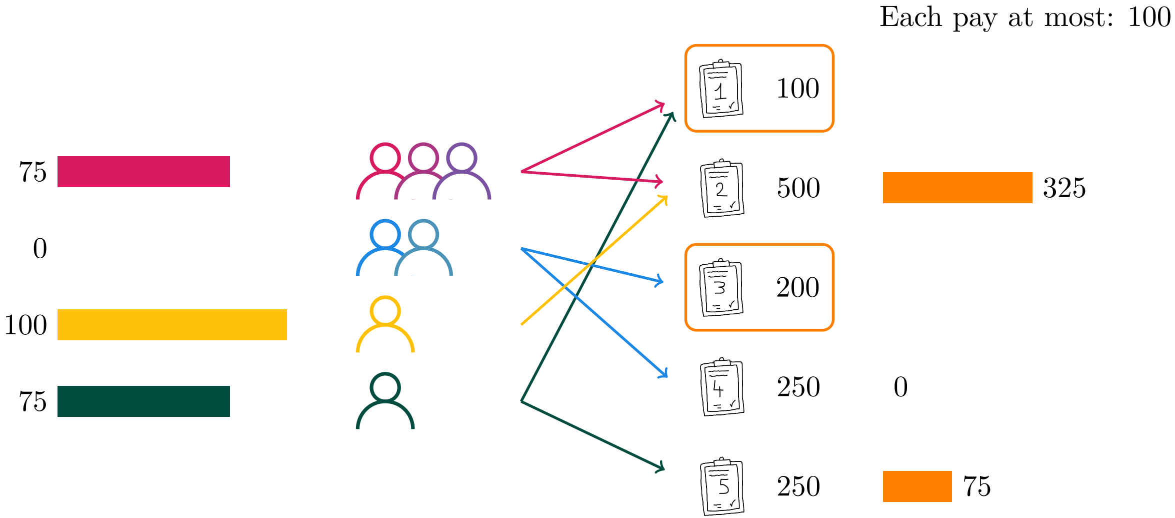
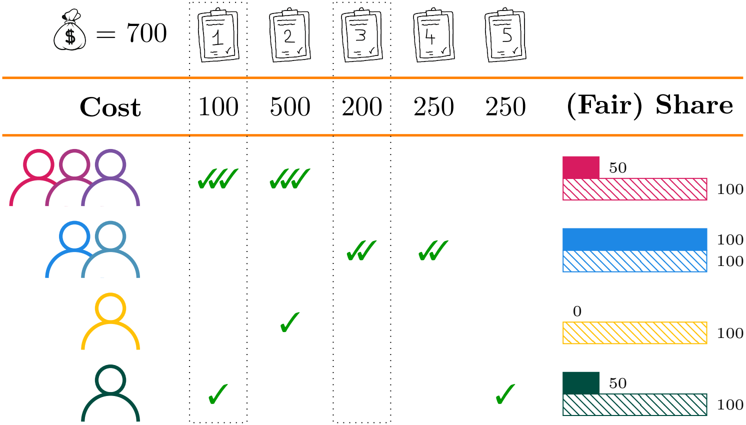

Each pay at most: 100
75
0
100
75
1
100
2
500
325
3
200
4
250
0
5
250
75

Rule X Satisfies Local Fair Share



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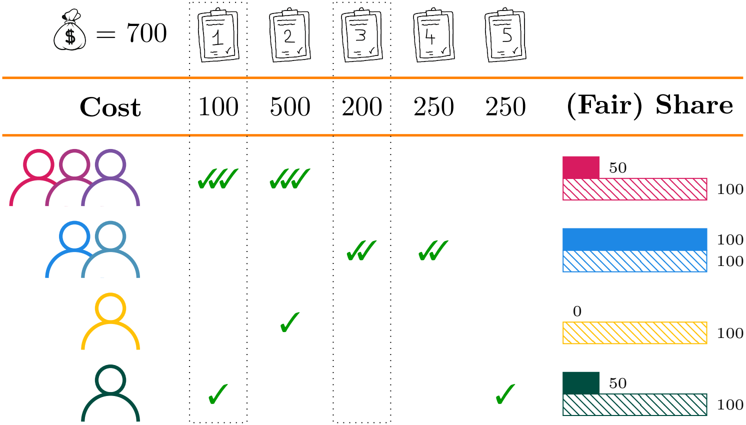

“Proof”: Before the first round at which not all agents pays in full the selected project, the share of an agent is equal to their money spent. Then, for every non-selected project, selecting it would provide a fair share to the agent who could no longer contribute in full to the project.

5. Justified Share

Cohesive Groups

New idea: I want to provide what is deserved by the agents! But **what** do they deserve and **who**?
↳ Cohesive groups deserve to be represented to the amount of budget they control!

Agents in $N \subseteq \mathcal{N}$ are *P-cohesive*, if

$$\underbrace{P \subseteq \bigcap_{i \in N} A_i}_{\text{They are similar}} \quad \text{and} \quad \underbrace{\frac{|N|}{n} \geq \frac{c(P)}{b}}_{\text{They control enough units of budget}}$$

Providing Agents What They Deserve

In any budget allocation the members of P should deserve the share they have in P: that's *Extended Justified Share* (EJS).

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$= 2$	1	2	3
Cost	1	1	1
	✓		
	✓	✓	
	✓		✓
		✓	✓

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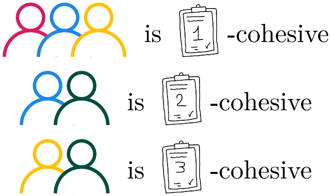
In any budget allocation the members of P should deserve the share they have in P: that's *Extended Justified Share* (EJS).

= 2			
Cost	1	1	1
	✓		
	✓	✓	
	✓		✓
		✓	✓

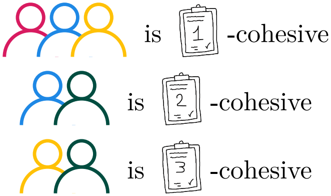
Cohesive Groups



In any budget allocation the members of P should deserve the share they have in P: that's **Strong** Extended Justified Share (EJS).

= 2	1	2	3
Cost	1	1	1
	✓		
	✓	✓	
	✓		✓
		✓	✓

Cohesive Groups



↳ Strong EJS is unsatisfiable!

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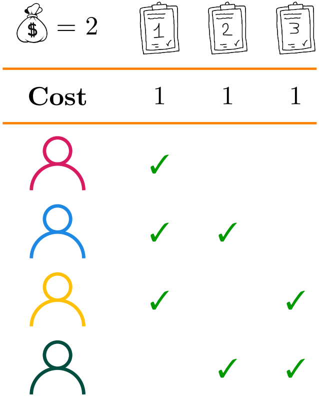
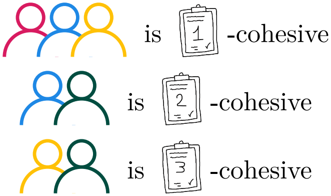
In any budget allocation *at least one of* the members of P should deserve the share they have in P: that's Extended Justified Share (EJS).



Cohesive Groups



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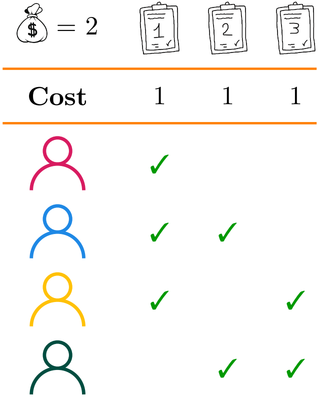
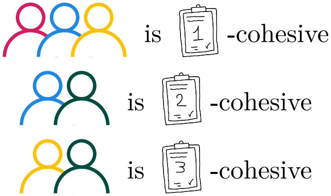


Cohesive Groups



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↳ EJS is satisfiable, stay tuned!

Achieving Extended Justified Share

A simple procedure that always return a feasible budget allocation satisfying EJS:

While there exists a P-cohesive group N, for any P:

- Choose (P, N) where N is P-cohesive that maximizes $\max_{i \in N} share(P, i)$;
- Select the projects from P;
- Remove agents in N.

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Computational perspective:

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- This runs in exponential time, but in FPT *time in the number of projects*;
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↳ Let's look for requirements that can be satisfied in polynomial time.

A weakening of EJS:

For every P-cohesive group N, there exist an agent $i \in N$ *for which there exists a project* $p \in \mathcal{P}$ such that:

$$share(\pi \cup \{p\}, i) \geq share(P, i).$$

A weakening of EJS:

For every P-cohesive group N, there exist an agent $i \in N$ *for which there exists a project* $p \in \mathcal{P}$ such that:

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This property is satisfied by Rule X, the proof is a simple adaptation from the one by Peters et al. (2021) showing that Rule X satisfies other representation requirements.

Towards Tractable Rules — Extended Justified Share up to One Project

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↳ But we can go further than that!

Towards Tractable Rules — Local Extended Justified Share

For *no* P-cohesive group N would *there exist a project* $p \in P \setminus \pi$ such that for all agent $i \in N$, we have:

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Local-EJS $\Rightarrow$ EJS-X: Let $i^\star$ be an agent with maximal share in N. By Local-EJS, for every $p \in P \setminus \pi$, there exist $i_p \in N$ such that:

$$\mathit{share}(\pi \cup \{p\}, i^\star) \geq \mathit{share}(\pi \cup \{p\}, i_p) \geq \mathit{share}(P, i_p) = \mathit{share}(P, i^\star).$$

A Partly Satisfying Result

Rule X satisfies Local-EJS (or EJS-X)...but only for unit-cost instances.

The proof is way too technical to present it here, it is a matter of tracking carefully the share of the agents throughout a run of Rule X.

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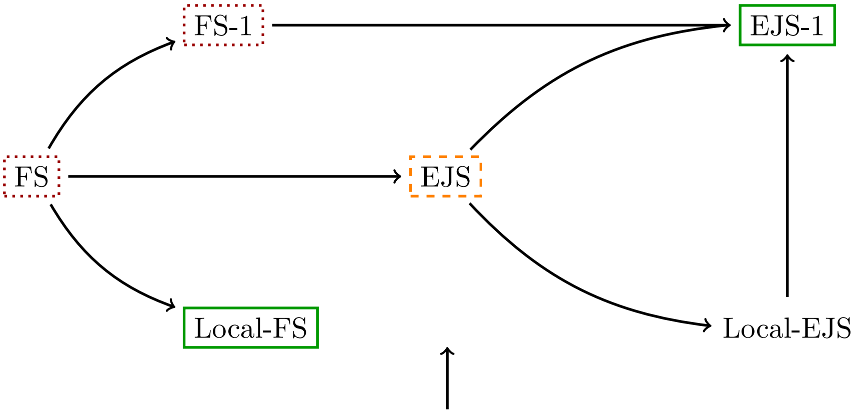
The proof is way too technical to present it here, it is a matter of tracking carefully the share of the agents throughout a run of Rule X.

Can we do better? Not with Rule X: we have a counterexample for Rule X in general PB instances; But there might be another rule out there (or Local-EJS cannot be satisfied in polynomial time)!

6. Conclusion

The Picture so Far



The arrow is proved to be missing here

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