Almost Group Envy-free Allocation of Indivisible Goods and Bads

Simon Rey, joint work with Haris Aziz

November 4, 2019

1. Fair division

Fair division

Introduction

Fundamental question

Given a *set of items* and a *set of agents* expressing *preferences* over the items, how can we *allocate* the items to the agents in the *fairest* way?

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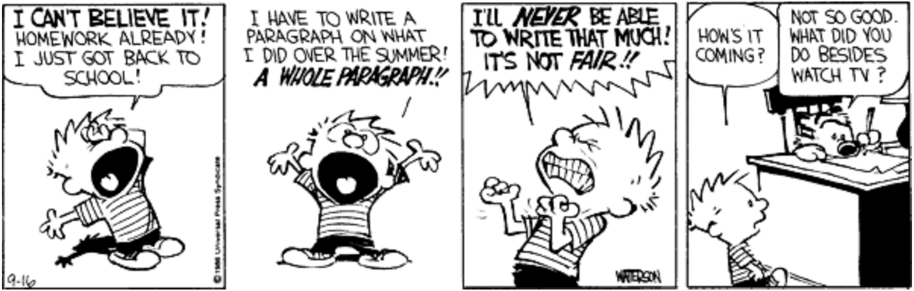
- *Fairness*: no one believes someone has been spoiled

Potential applications - Task sharing



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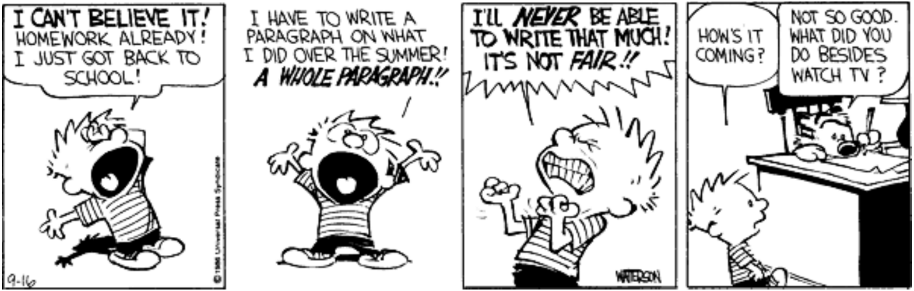

- *Set of items*: a set of tasks

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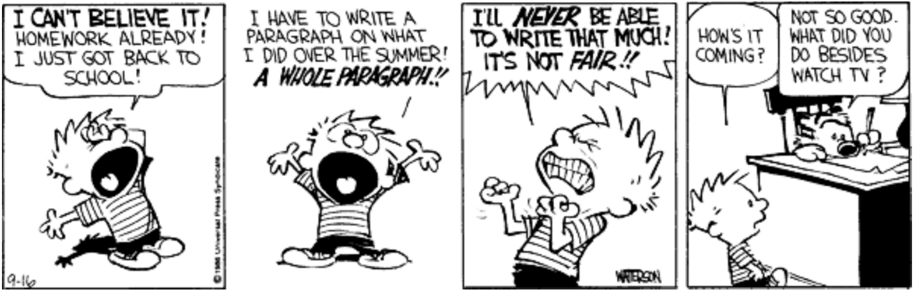

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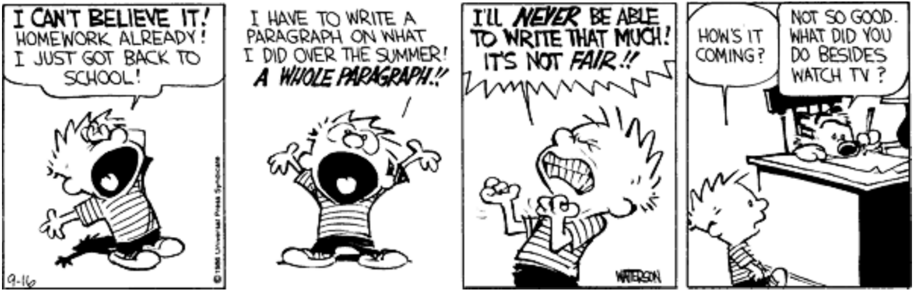

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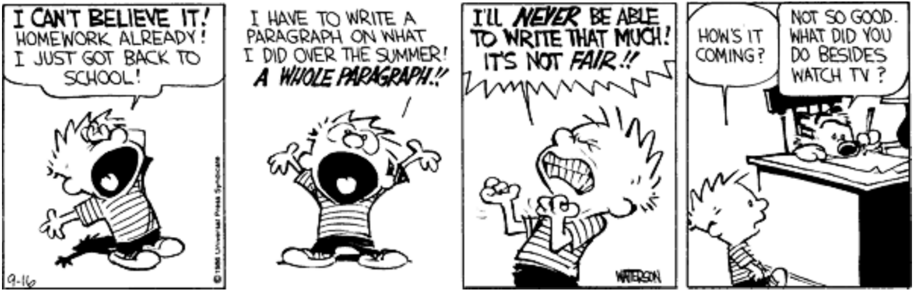

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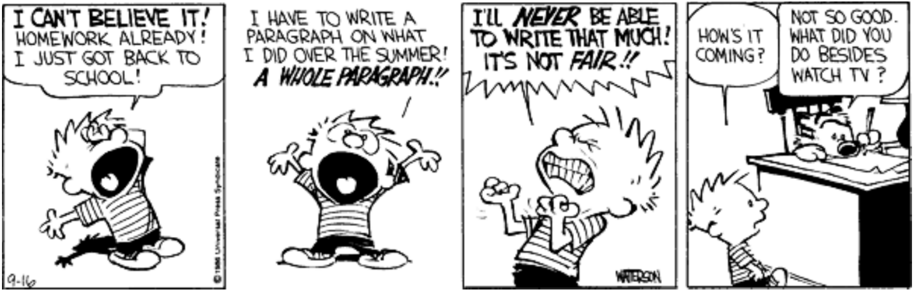

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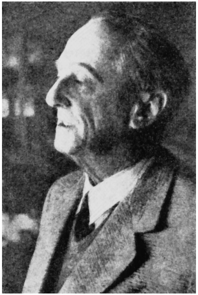
- *Fairness*: (almost) no jealousy between agents

Fair division

Historical perspectives

Seminal work



CONTRIBUTED PAPERS, II

Wednesday, Sept. 17, 9:30 A.M. Chairman: HERMAN O. A. WOLD (Sweden)

THE PROBLEM OF FAIR DIVISION

HUGO STEINHAUS, Professor, University of Warsaw (Poland)

There is a custom, probably many centuries old, of dividing an object into two equal parts by letting one partner halve it and the other choose his half. The custom is also that the older one has to halve.

This procedure is a *fair game*. As every game, it has its *rules* and its *methods*. The rule for the first partner allows him to cut the object—it may be a cake—as he pleases; the rule for the second one gives him a free choice of one of the two slices. The methods are: (1) for the older

Hugo Steinhauss defined in 1948 *The Fair Division Problem*, he introduced formally the *cake-cutting* problem and proposed some allocation rules.

Fair division's grounds

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General Equilibria

Gérard Debreu

Kenneth Arrow

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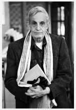
Fair Division

Game Theory

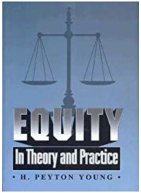

John von Neumann

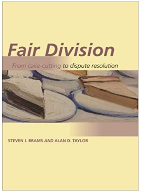

Oscar Morgenstern

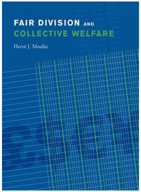

John Nash

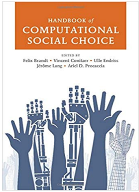

Lloyd Shpaley

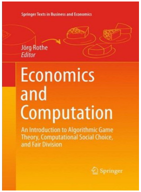
A prolific literature

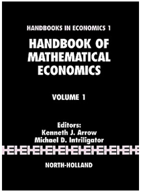


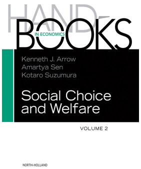


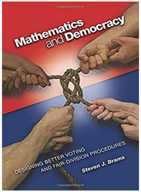


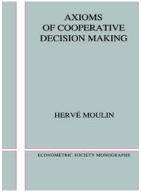


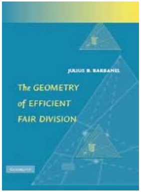












Today's topic

- *Set of items*:

Today's topic

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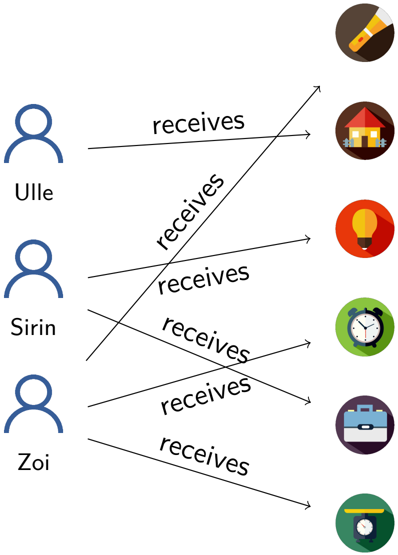
2. Model


Ulle
Sirin
Zoi

Fair division of indivisible items



<u>DEFINITION</u>: ALLOCATION

Let $\mathcal{N}$ be a set of agents, $\mathcal{O}$ an set of items, an allocation $\pi = \langle \pi_1, \ldots, \pi_{|N|} \rangle$ from $\mathcal{O}$ to $\mathcal{N}$ is a vector which i's component corresponds to the subset of items allocated to agent i. An allocation is such that:

- all the items are allocated, and,
- an item can only be allocated to one agent.

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Agent $i \in \mathcal{N}$ expresses her preferences through an *utility function*:

$$u_i : 2^{\mathcal{O}} \rightarrow \mathbb{R}.$$

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For a given agent $i \in \mathcal{N}$, an item $o \in \mathcal{O}$ is said to be:

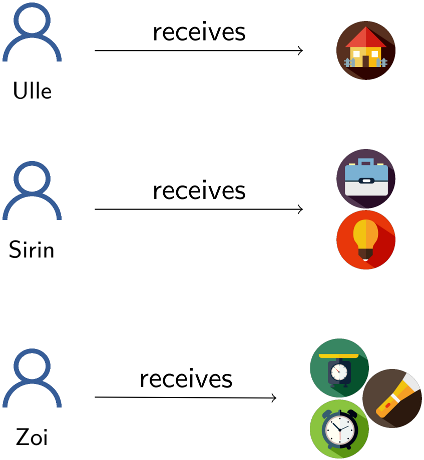
- a *good* if $u_i(o) \geq 0$,
- a *bad* if $u_i(o) \leq 0$.

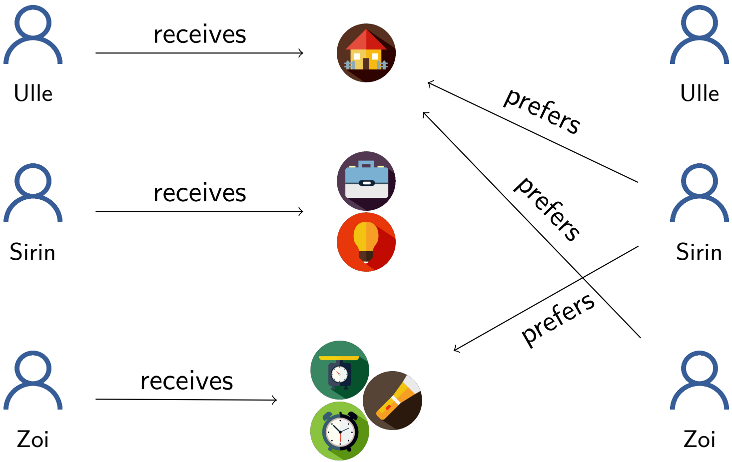
3. No-envy as a fairness criteria

No-envy as a fairness criteria

When there are only goods

Envy-freeness (EF)

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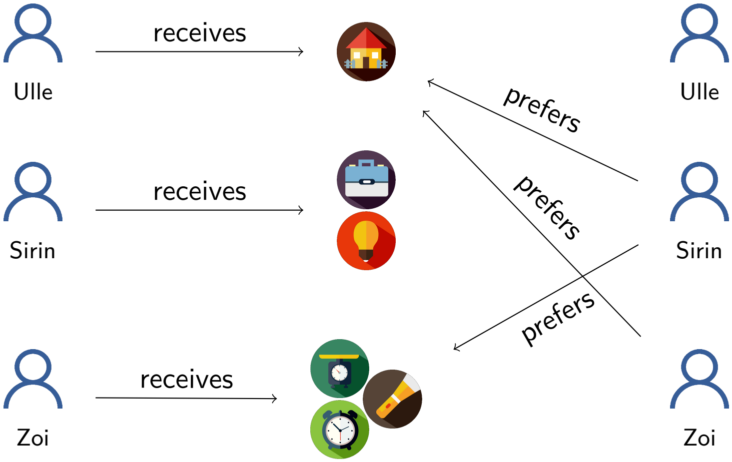

[1] Foley "Resource allocation and the public sector." (1967)

Summary on envy-freeness

✗ (
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Sirin: 
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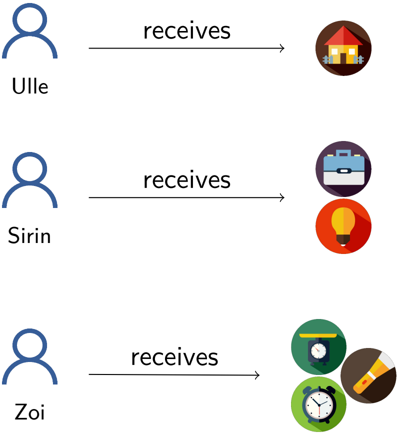
➥ How can we get existence guarantee ?

Envy-freeness up to one good (EF1)



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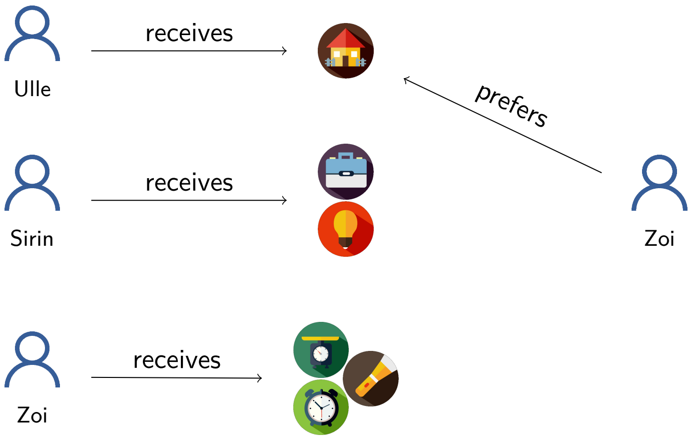

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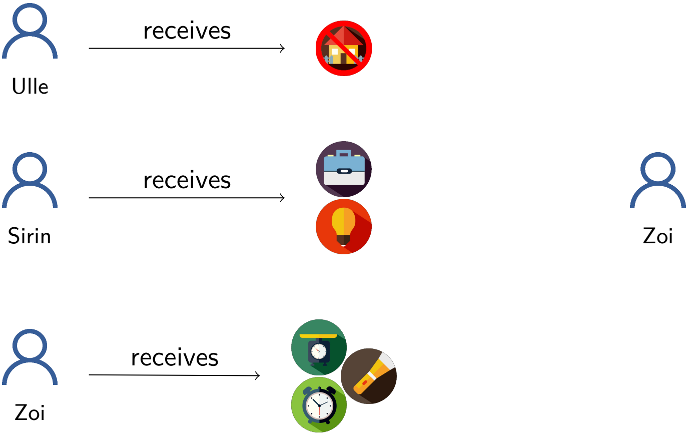

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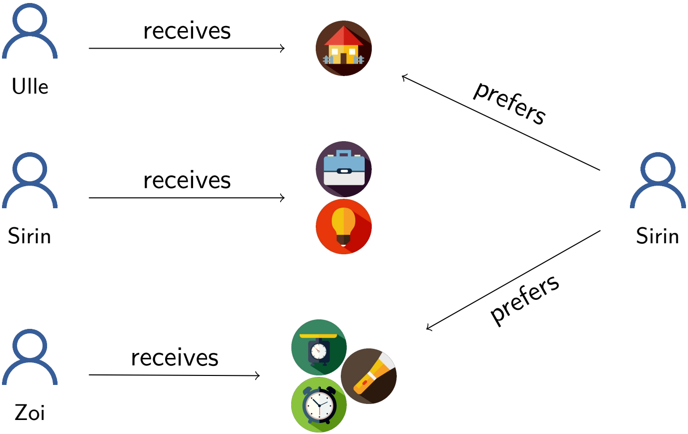

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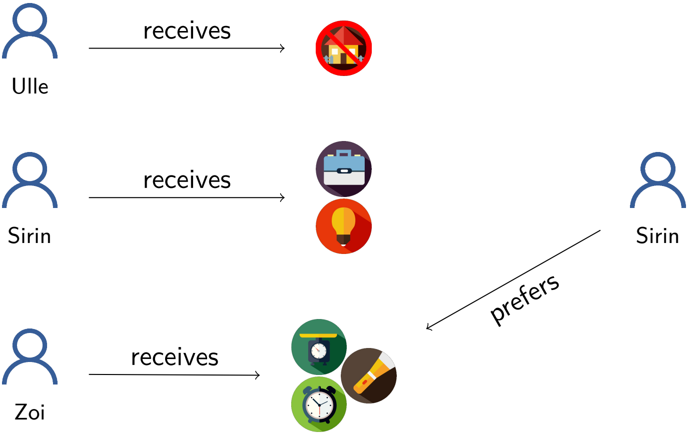

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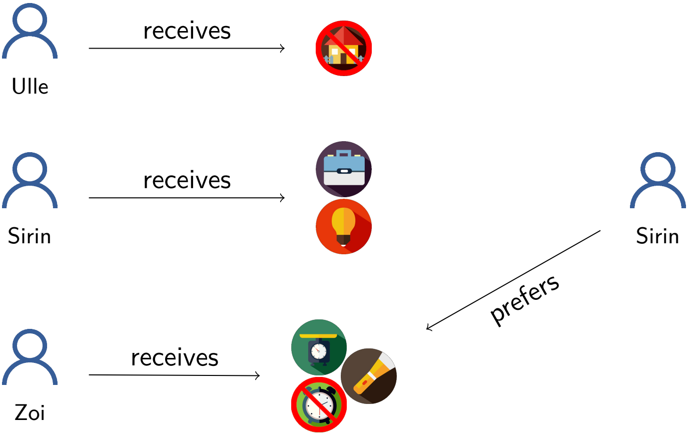

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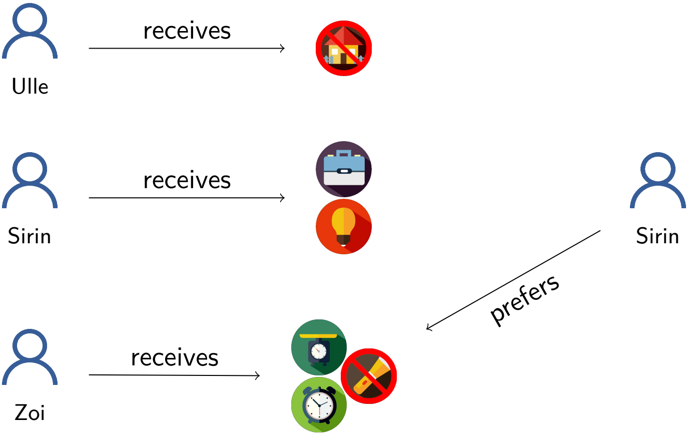

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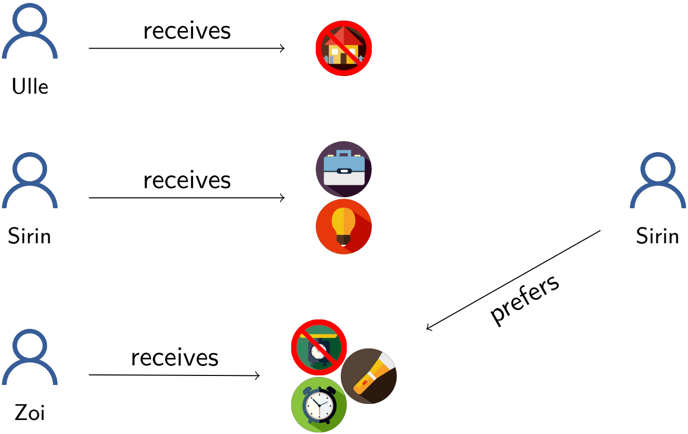

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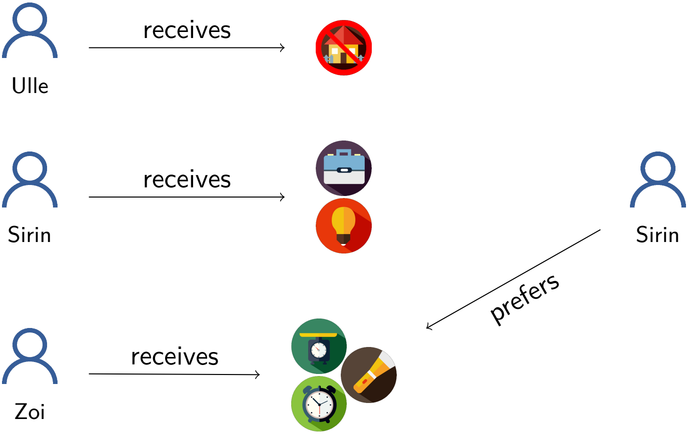
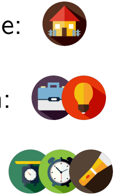


[3] Budish "The combinatorial assignment problem: Approximate competitive equilibrium from equal incomes" (2011)

Summary on envy-freeness up to one good

✗ (Ulle: / Sirin: / Zoi:) is not EF1: Sirin is still envious after removal.



[2] Lipton, Markakis, Mossel, and Saberi "On approximately fair allocations of indivisible goods" (2004)

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✓ An envy-free up to one good allocation always exists and can be computed efficiently (in polynomial time) even for non-additive preferences. [2]

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➥ What does happen when there are *bads*?

---

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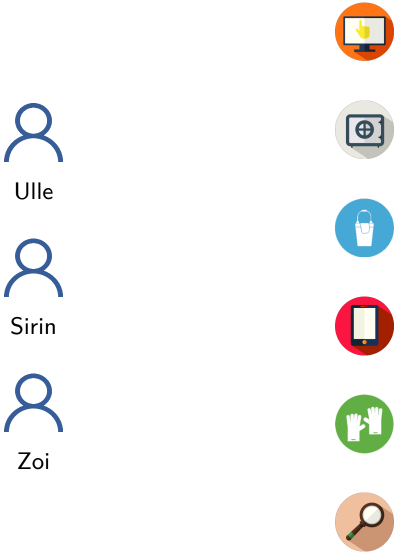
No-envy as a fairness criteria

When there are goods and bads

Fair division of indivisible goods and bads



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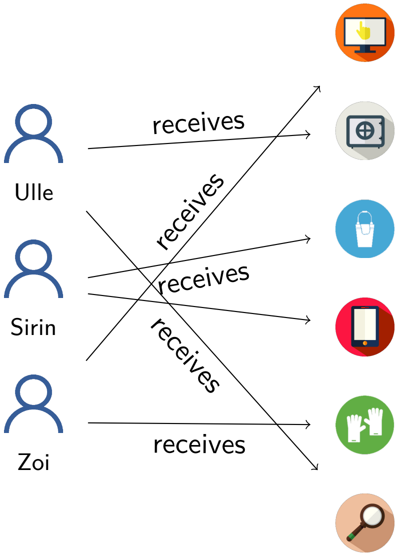

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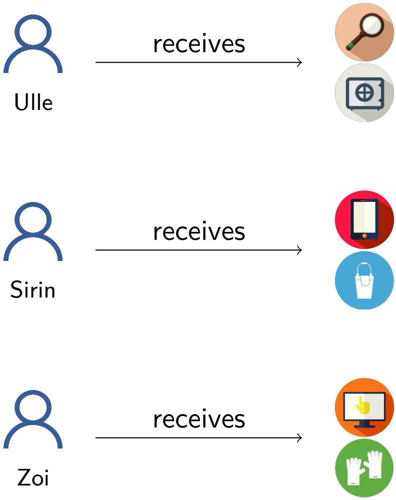

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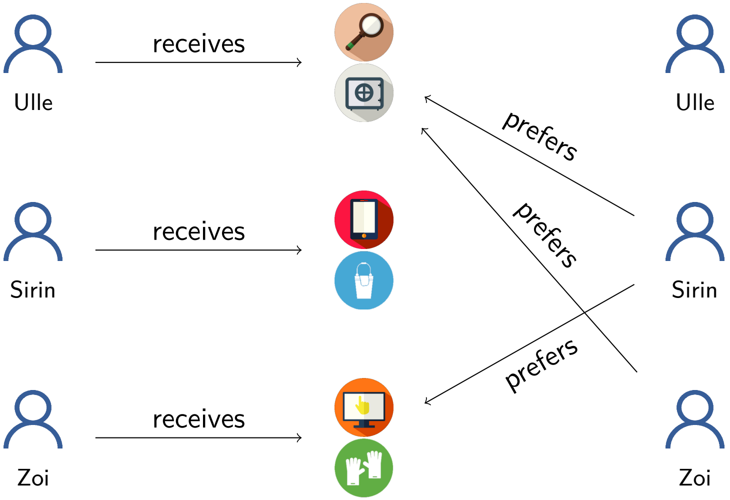
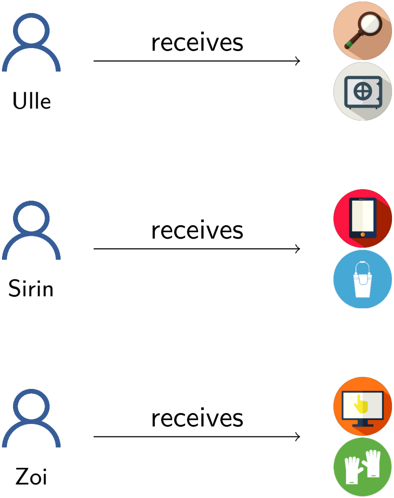
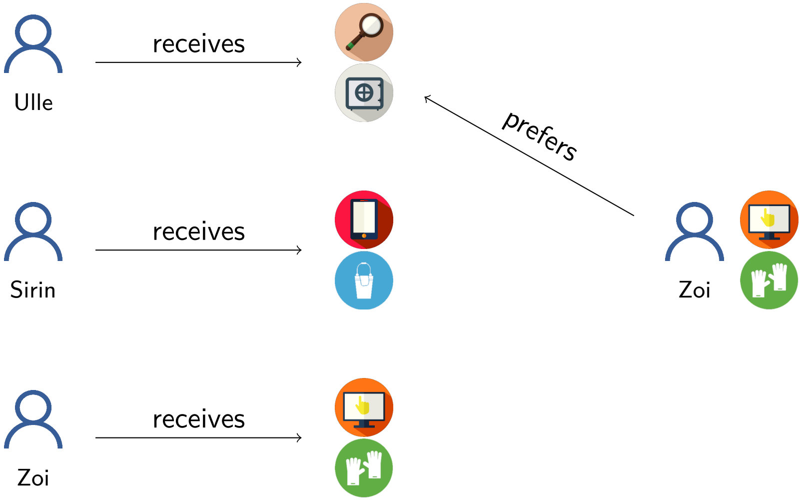


Envy-freeness up to one item (EF1)

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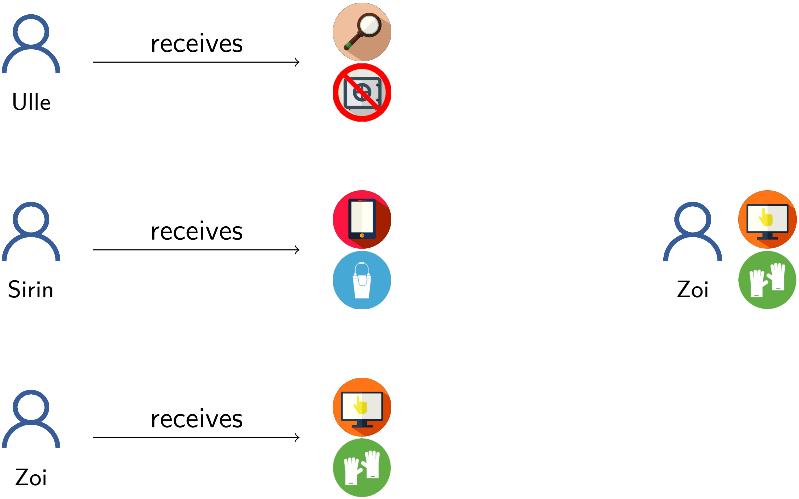

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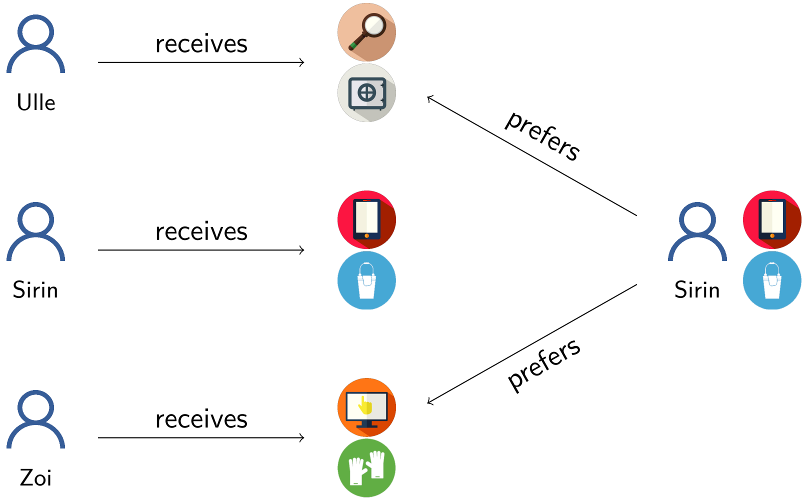

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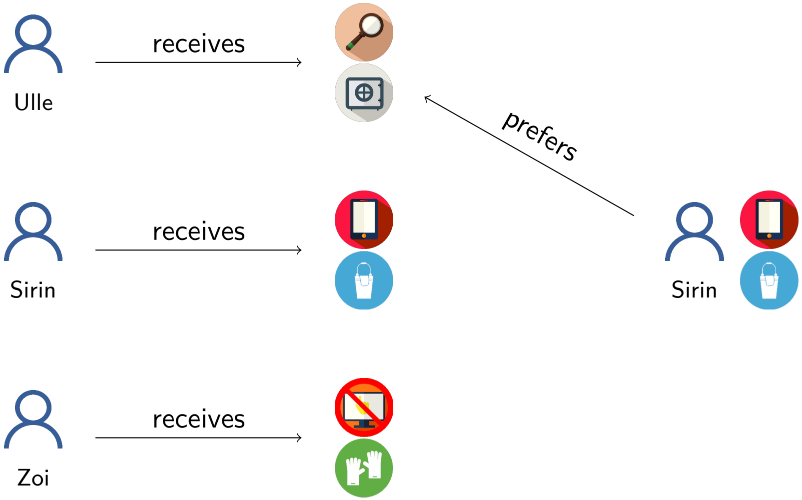

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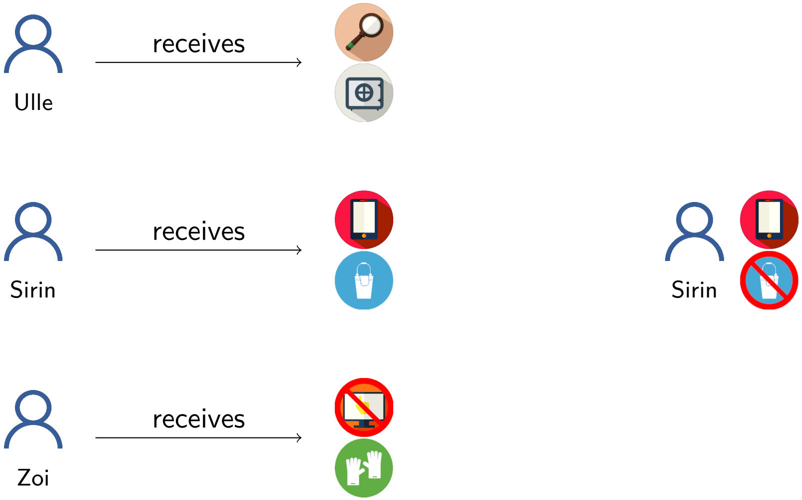

Summary on envy-freeness up to item

✓

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Sirin: 

Zoi: 

is EF1.

[4] Aziz, Caragiannis, Igarashi, and Walsh “Fair allocation of combinations of indivisible goods and chores” (2019)

Summary on envy-freeness up to item

✓ (Ulle:, Sirin:, Zoi:) is EF1.

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➥ How can we generalize this approach to *groups of agents* ?

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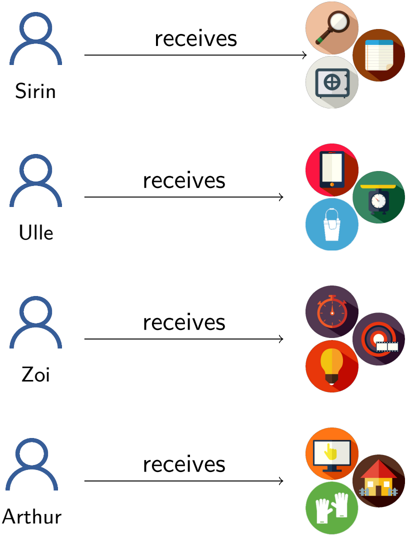
4. Group envy-free allocations

Group envy-free allocations (GEF)



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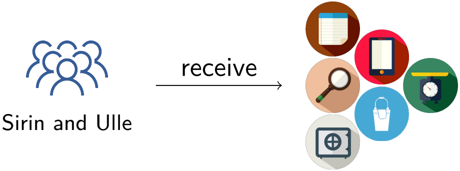

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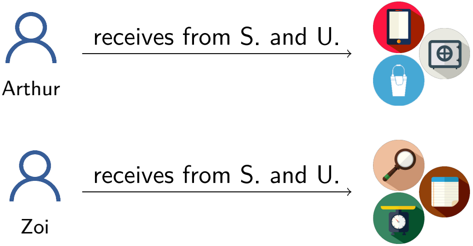
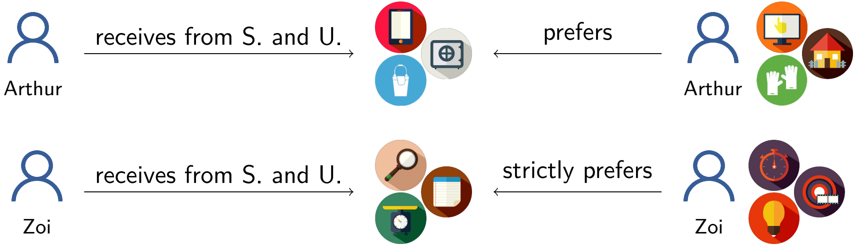


Group envy-free allocations (GEF)

What does "prefer" mean here?



Mathematical formalization

DEFINITION: GROUP ENVY-FREENESS

Let $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}} \rangle$ be an instance with both goods and bads.

An allocation $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ is GEF if:

1. for every non-empty $S \subseteq \mathcal{N}$ and $T \subseteq \mathcal{N}$ such that $|S| = |T|$
2. there is no reallocation $\pi' \in \Pi(\pi_T, S)$ such that:
3. π' Pareto-dominates π for agents in T.

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✓ Implies Pareto-optimality for $S = T = \mathcal{N}$.

✓ Implies envy-freeness for $|S| = |T| = 1$,

✗ which implies non-existence.

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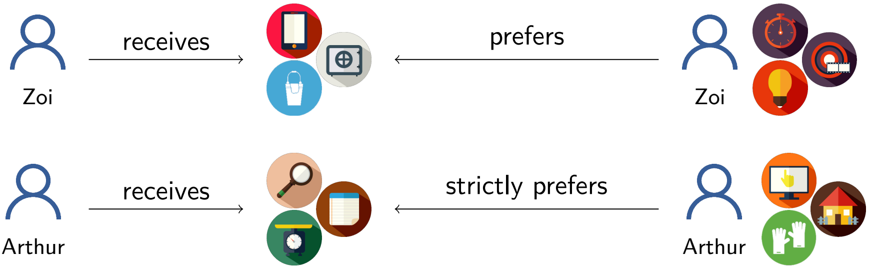
➥ How can we get guarantees for existence of GEF allocations?

Group envy-freeness up to one item



Group envy-freeness up to one item

Group envy-freeness up to one item

Need to check for every reallocation.

Mathematical formalization

<u>Definition</u>: GEF1

Let $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}} \rangle$ be an instance with both goods and bads.

An allocation $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ is GEF1 if:

1. for every pair of non empty groups, $S \subseteq \mathcal{N}$ and $T \subseteq \mathcal{N}$, such that $|S| = |T|$,
2. for every reallocation $\pi' \in \Pi\left(\pi_T, S\right)$, and
3. for every agent $i \in S$, there exists $o_i \in \pi_i \cup \pi'_i$ such that

$\langle u_i(\pi'_i \backslash \{o_i\}) \rangle_{i \in S}$ does not Pareto-dominate $\langle u_i(\pi_i \backslash \{o_i\}) \rangle_{i \in S}$.

Mathematical formalization

Definition: GEF1

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✓ Implies envy-freeness up to one item for $|S| = |T| = 1$,

Related work

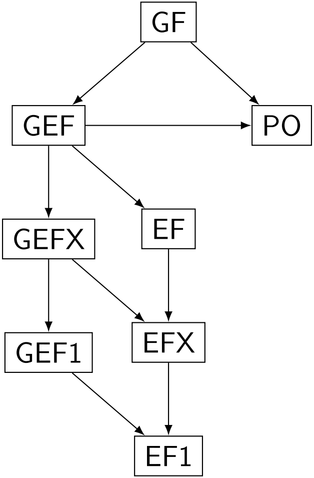
[5] *1972* Schmeidler and Vind: introduced the idea of fairness criteria between groups of different size.

[6] *1992* Berliant, Thomson, and Dunz: defined *group envy-freeness* when items are divisible goods.

[7] *2019* Conitzer, Freeman, Shah, and Vaughan: extended group-envy freeness to *group fairness* when items are indivisible goods and groups of different size are compared.

[5] Schmeidler and Vind "Fair net trades" (1972)
[6] Berliant, Thomson, and Dunz "On the fair division of a heterogeneous commodity" (1992)
[7] Conitzer, Freeman, Shah, and Vaughan "Group Fairness for the Allocation of Indivisible Goods" (2019)

Taxonomy of fairness criteria



5. Existence results for GEF1 allocations

When all items are goods

When there are only goods, [7] shown that by maximizing the *Nash welfare* one get GEF1.

THEOREM: CONITZER, FREEMAN, SHAH, AND VAUGHAN

Let $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}} \rangle$ be an instance *with only goods*, and such that preferences are *additives*. An allocation $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ satisfying GEF1 *always exists* and can be computed in *pseudo-polynomial time*.

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➥ Can we generalized this result for goods and bads ?

---

[7] Conitzer, Freeman, Shah, and Vaughan "Group Fairness for the Allocation of Indivisible Goods" (2019)

Existence results for GEF1 allocations

With identical preferences

GEF1 with identical preferences

LEMMA:

Let $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}} \rangle$ be an instance with goods and bad such that the preferences are *identical and additive*. Every allocation $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ satisfying *EFX* also satisfies *GEF1*.

Egal-sequential algorithm

Lemma:

An *EFX* allocation can be computed in time in $\mathcal{O}(mn)$.

Egal-sequential algorithm

Input: An instance $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}} \rangle \in \mathcal{I}$ such that
$\forall i \in \mathcal{N}, u_i = u$, for a given utility function u

Output: $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ an EFX allocation

$\pi \leftarrow$ empty allocation

Sort items $o_1, \ldots, o_m$ in decreasing order of $|u(o)|$

for $j = 1$ to m **do**

  if $u(o_j) \geq 0$ **then**

    Choose $i^* \in \arg\min_{i \in \mathcal{N}} u(\pi_i)$

  else

    Choose $i^* \in \arg\max_{i \in \mathcal{N}} u(\pi_i)$

  Allocate o_j to i^*: $\pi_{i^*} \leftarrow \pi_{i^*} \cup \{o_j\}$

return π

GEF1 with identical preferences

<u>THEOREM</u>:

Let $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}} \rangle$ be an instance with goods and bad such that the preferences are *identical and additive*. An allocation $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ satisfying GEF1 *always exists* and can be computed in *polynomial time*.

Example of the egal-sequential algorithm

EXAMPLE: Let us consider two agents with the following preferences

					
5	-4	3	-6	1	2

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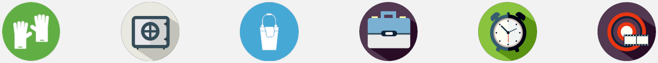

-6 5 -4 3 2 1

The current allocation is:

Sirin: $\emptyset$ Ulle: $\emptyset$

Utility : 0 Utility : 0

Example of the egal-sequential algorithm

EXAMPLE: Let us consider two agents with the following preferences

-6 5 -4 3 2 1

The current allocation is:

Sirin: 

Utility : -6

Ulle: $\emptyset$

Utility : 0

Example of the egal-sequential algorithm

<u>EXAMPLE</u>: Let us consider two agents with the following preferences

-6 5 -4 3 2 1

The current allocation is:

Sirin: 

Utility : -1

Ulle: $\emptyset$

Utility : 0

Example of the egal-sequential algorithm

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The current allocation is:

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Example of the egal-sequential algorithm

EXAMPLE: Let us consider two agents with the following preferences

-6	5	-4	3	2	1

The current allocation is:

Sirin: 

Utility : 0

Ulle: 

Utility : 1

Existence results for GEF1 allocations

With ternary symmetric preferences

Ternary symmetric preferences

Definition:

An agent $i \in \mathcal{N}$ has ternary symmetric preferences if her preferences are *additive* and there exists $\alpha_i \in \mathbb{R}_{>0}$ such that:

$$\forall o \in \mathcal{O}, u_i(o) \in \{-\alpha_i, 0, \alpha_i\}.$$

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Example:

						
Ulle	1	-1	1	-1	0	1
Sirin	3	-3	0	-3	-3	3
Arthur	-5	-5	5	-5	-5	5

GEF1 with ternary symmetric preferences

<u>Lemma</u>:

Let $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}} \rangle$ be an instance with goods and bad such that the preferences are *ternary symmetric*. Every allocation $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ that is *leximin-optimal* also satisfies *GEF1*.

The ternary flow algorithm

Lemma:

An allocation *leximin-optimal* can be computed in *polynomial time*.

The ternary flow algorithm

Input: An instance $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}}) \rangle$ where preferences are ternary symmetric.

Output: $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ an leximin-optimal allocation

$O^+ = \{o \in \mathcal{O} : \max_i u_i(o) > 0\}$

$O^0 = \{o \in \mathcal{O} : \max_i u_i(o) = 0\}$

$O^- = \{o \in \mathcal{O} : \max_i u_i(o) < 0\}$

Consider new utilities: $\forall i \in \mathcal{N}, \forall o \in O^+, u'_i(o) = \begin{cases} 1 \text{ si } u_i(o) = 1, \\ 0 \text{ sinon} \end{cases}$

$\pi \leftarrow$ result of the Nash flow algorithm on $\langle \mathcal{N}, O^+, (u'_i)_{i \in \mathcal{N}} \rangle$.

for $o \in O^-$ **do**

  Allocate o to $i^* \in \arg\max_{i \in \mathcal{N}} u(\pi_i)$

for $o \in O^0$ **do**

  Allocate o to an agent i^* such that $u_{i^*}(o) = 0$.

return π

GEF1 with ternary symmetric preferences

<u>THEOREM</u>:

Let $I = \langle \mathcal{N}, \mathcal{O}, (u_i)_{i \in \mathcal{N}} \rangle$ be an instance with goods and bad such that the preferences are *ternary symmetric*. An allocation $\pi \in \Pi(\mathcal{O}, \mathcal{N})$ satisfying GEF1 *always exists* and can be computed in *polynomial time*.

Example of the ternary flow algorithm

EXAMPLE: Let us consider three agents with the following preferences:

						
Ulle	1	-1	1	-1	0	1
Sirin	3	-3	0	-3	-3	3
Arthur	-5	-5	5	-5	-5	5

We have then :

O^+:  O^0:  O^-: 

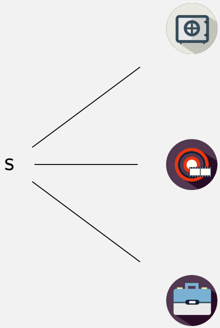
<u>EXAMPLE</u>: Consider the following new preferences for items in O^+ used for the Nash flow algorithm [8] :

			
Ulle	1	1	1
Sirin	1	0	1
Arthur	0	1	1

[8] Darmann and Schauer "Maximizing Nash product social welfare in allocating indivisible goods" (2015)

Example of the ternary flow algorithm - Nash flow

Example:



Example of the ternary flow algorithm - Nash flow

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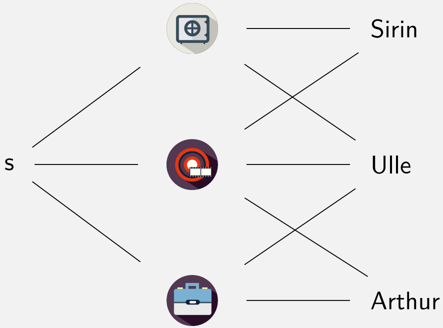

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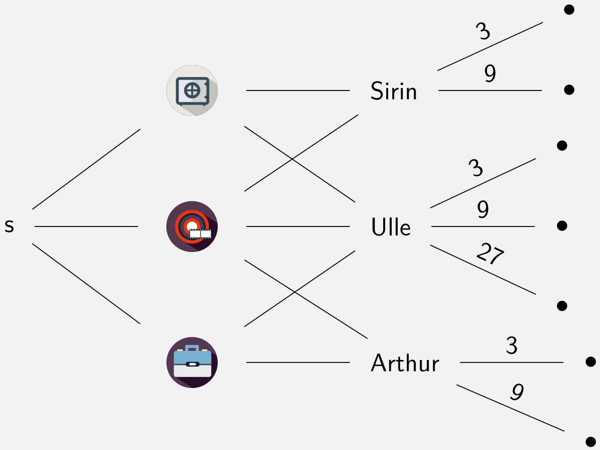

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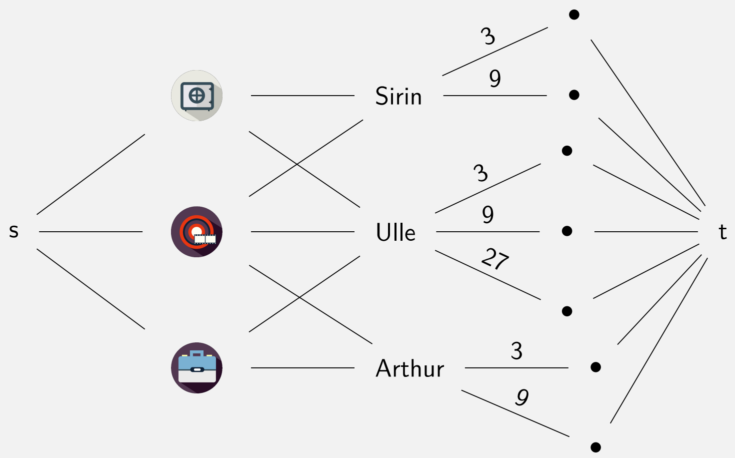

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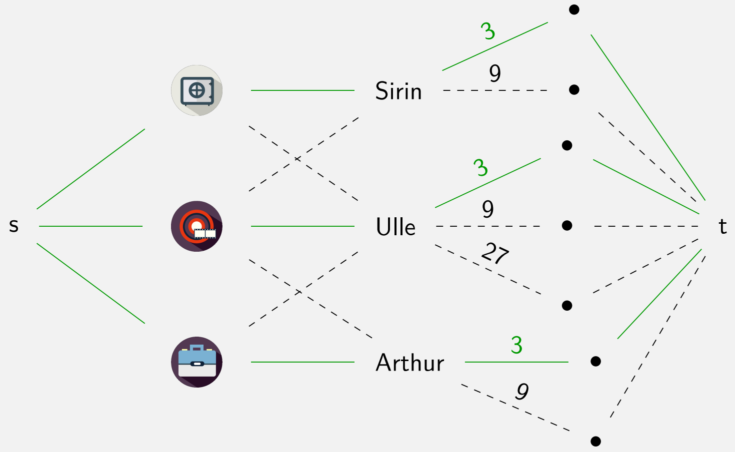


EXAMPLE: The partial allocation obtained is:

Sirin: 

Utility : 3

Arthur: 

Utility : 5

Ulle: 

Utility : 1

Example of the ternary flow algorithm - Final allocation

<u>EXAMPLE</u>: The partial allocation obtained is:

Sirin:  Arthur:  Ulle: 

Utility : 3 Utility : 5 Utility : 1

We allocate the final items to get:

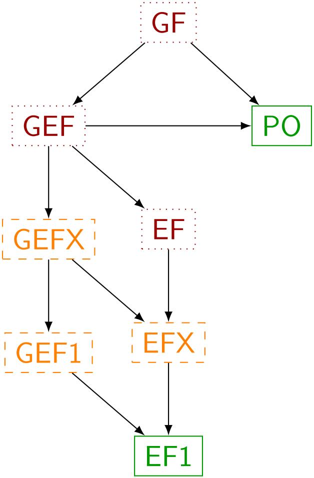
Sirin:  Arthur:  Ulle: 

Utility : 0 Utility : 0 Utility : 1

Existence results for GEF1 allocations

Summary on existence results

Taxonomy of fairness criteria with existence results



6. Conclusion

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We have...

- ... introduced a new fairness criteria for groups of agents when there are goods and bads,
- ... presented its links with common fairness criteria,
- ... given some existence results on classical restricted domains.

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- ... introduced a new fairness criteria for groups of agents when there are goods and bads,
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We still need to ...

- ... find a general result for GEF1 with additive preferences,
- ... study the "up to any item" relaxation of GEF, namely GEFX, with the perspective of EFX,
- ... extend other fairness criteria to groups of agents.

Conclusion

The End