Let's Agree to Agree: Targeting Consensus for Incomplete Preferences through Majority Dynamics

Simon Rey, together with Sirin Botan and Zoi Terzopoulou

IJCAI 2022

1. Introduction

Deciding for an Online Platform

Deciding for an Online Platform

We have never used Bridge
AppEar is better than C-nnect

Deciding for an Online Platform

We have never used Bridge
AppEar is better than C-nnect

We have never used C-nnect
Bridge is more stable than AppEar

Deciding for an Online Platform





We have never used Bridge
AppEar is better than C-nnect

$a \quad b$
$\downarrow$
c

$a \quad b$
$\downarrow$
c

We have never used C-nnect
Bridge is more stable than AppEar

$b \quad c$
$\downarrow$
a

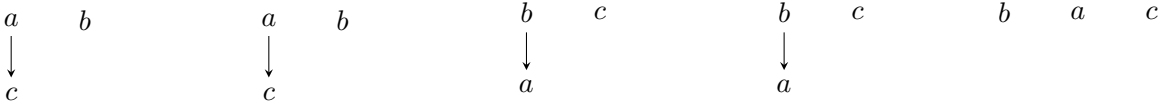
$b \quad c$
$\downarrow$
a

No opinion

$b \quad a \quad c$

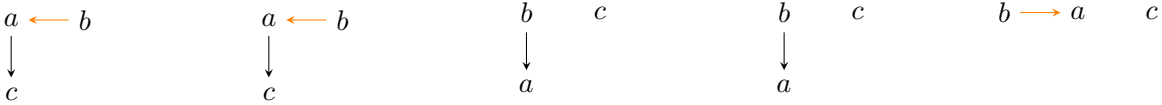
Deciding for an Online Platform

The committee meets to discuss the alternatives and starts by comparing AppEar and Bridge



Deciding for an Online Platform

The committee meets to discuss the alternatives and starts by comparing AppEar and Bridge

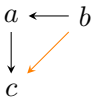


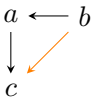
Deciding for an Online Platform

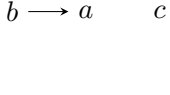




The committee meets to discuss the alternatives and starts by comparing AppEar and Bridge





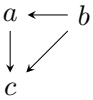


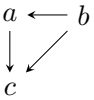
Deciding for an Online Platform

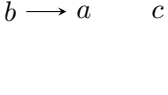




The merits of Bridge over C-nnect are then discussed





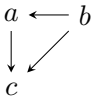


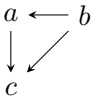
Deciding for an Online Platform

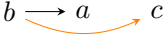




The merits of Bridge over C-nnect are then discussed





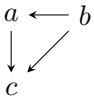


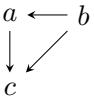
Deciding for an Online Platform

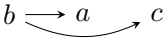




Finally, AppEar and C-nnect are compared





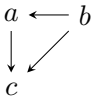


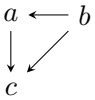
Deciding for an Online Platform





Finally, AppEar and C-nnect are compared



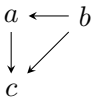


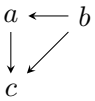
Deciding for an Online Platform

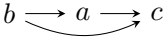




There is nothing more to discuss at this point



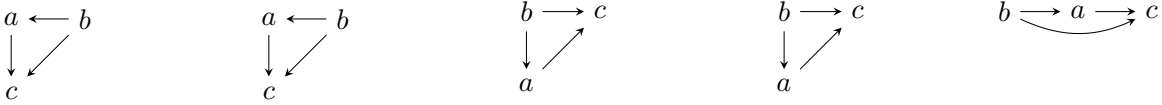




↳ Note the existence of an obvious consensual alternative now: Bridge.

Deciding for an Online Platform

There is nothing more to discuss at this point



↳ Note the existence of an obvious consensual alternative now: Bridge.

Our goal is to study this dynamic process!

The Majority Dynamic

Let $\sigma = (p_1, \ldots, p_\ell)$ be an *update order* over ordered pairs of alternatives.

Starting from an incomplete profile P, pairs are discussed following σ.

The Majority Dynamic

Let $\sigma = (p_1, \ldots, p_\ell)$ be an *update order* over ordered pairs of alternatives.

Starting from an incomplete profile P, pairs are discussed following σ.

When the pair ab is discussed at time t, every agent's partial order $\succ_i^{t-1}$ is updated such that:

The Majority Dynamic

Let $\sigma = (p_1, \ldots, p_\ell)$ be an *update order* over ordered pairs of alternatives.

Starting from an incomplete profile P, pairs are discussed following σ.

When the pair ab is discussed at time t, every agent's partial order $\succ_i^{t-1}$ is updated such that:

$$\succ_i^t = \begin{cases} \succ_i^{t-1} & \text{if } ab \text{ or } ba \in \succ_i^{t-1}\text{: } \textit{no update if the pair is already ranked} \\ & \\ & \end{cases}$$

The Majority Dynamic

Let $\sigma = (p_1, \ldots, p_\ell)$ be an *update order* over ordered pairs of alternatives.

Starting from an incomplete profile P, pairs are discussed following σ.

When the pair ab is discussed at time t, every agent's partial order $\succ_i^{t-1}$ is updated such that:

$$\succ_i^t = \begin{cases} \succ_i^{t-1} & \text{if } ab \text{ or } ba \in \succ_i^{t-1}\text{: } \textit{no update if the pair is already ranked} \\ [\![\succ_i^{t-1} \cup \{ab\}]\!] & \text{if } N_{ab} > N_{ba}\text{: } \textit{a preferred to b if the majority prefers a over b} \end{cases}$$

$[\![\succ]\!]$ denotes the *transitive closure* of the order $\succ$.

The Majority Dynamic

Let $\sigma = (p_1, \ldots, p_\ell)$ be an *update order* over ordered pairs of alternatives.

Starting from an incomplete profile P, pairs are discussed following σ.

When the pair ab is discussed at time t, every agent's partial order $\succ_i^{t-1}$ is updated such that:

$$\succ_i^t = \begin{cases} \succ_i^{t-1} & \text{if } ab \text{ or } ba \in \succ_i^{t-1}\text{: } \textit{no update if the pair is already ranked} \\ [\![\succ_i^{t-1} \cup \{ab\}]\!] & \text{if } N_{ab} > N_{ba}\text{: } \textit{a preferred to b if the majority prefers a over b} \\ [\![\succ_i^{t-1} \cup \{ba\}]\!] & \text{if } N_{ab} < N_{ba}\text{: } \textit{b preferred to a if the majority prefers b over a} \end{cases}$$

$[\![\succ]\!]$ denotes the *transitive closure* of the order $\succ$.

The Majority Dynamic

Let $\sigma = (p_1, \ldots, p_\ell)$ be an *update order* over ordered pairs of alternatives.

Starting from an incomplete profile P, pairs are discussed following σ.

When the pair ab is discussed at time t, every agent's partial order $\succ_i^{t-1}$ is updated such that:

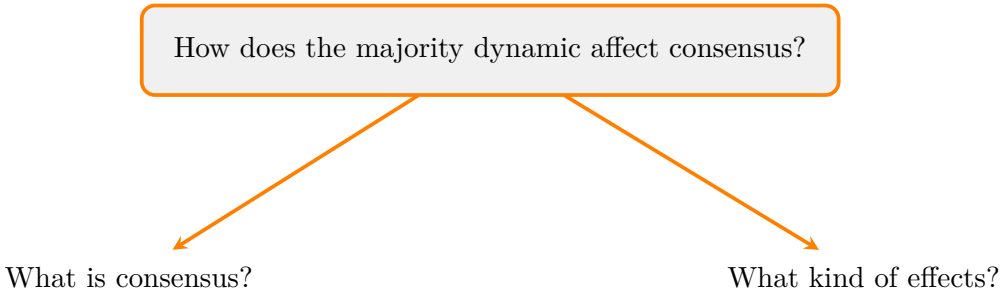
$$\succ_i^t = \begin{cases} \succ_i^{t-1} & \text{if } ab \text{ or } ba \in \succ_i^{t-1}\text{: } \textit{no update if the pair is already ranked} \\ [\![\succ_i^{t-1} \cup \{ab\}]\!] & \text{if } N_{ab} > N_{ba}\text{: } \textit{a preferred to b if the majority prefers a over b} \\ [\![\succ_i^{t-1} \cup \{ba\}]\!] & \text{if } N_{ab} < N_{ba}\text{: } \textit{b preferred to a if the majority prefers b over a} \\ [\![\succ_i^{t-1} \cup \{ab\}]\!] & \text{if } N_{ab} = N_{ba}\text{: } \textit{tie-breaking is determined by the order of the pair (ab here)} \end{cases}$$

$[\![\succ]\!]$ denotes the *transitive closure* of the order $\succ$.

Question of Interest

How does the majority dynamic affect consensus?

Question of Interest



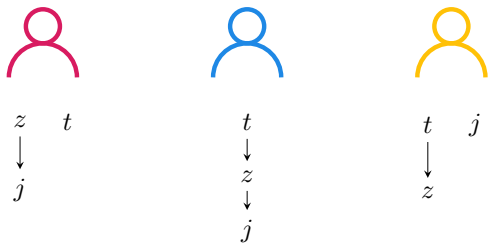
2. Preserving Condorcet Consensus

Condorcet Consensus

Condorcet Consensus: There exists an alternative *strictly* winning all pairwise majority contests against another alternative.

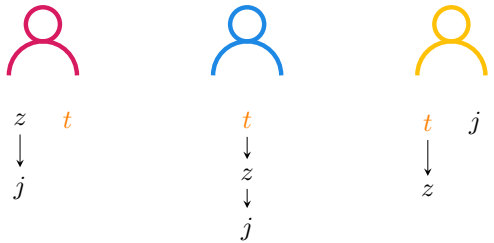
Condorcet Consensus

Condorcet Consensus: There exists an alternative *strictly* winning all pairwise majority contests against another alternative.



Condorcet Consensus

Condorcet Consensus: There exists an alternative *strictly* winning all pairwise majority contests against another alternative.



t against z: 2 for t 0 for z
t against j: 1 for t 0 for j

Preserving Condorcet Consensus

Preserving Consensus: For every profile, if there exists consensus initially, then for every update order, there will be consensus afterwards.

Preserving Condorcet Consensus

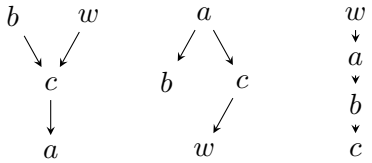
Preserving Consensus: For every profile, if there exists consensus initially, then for every update order, there will be consensus afterwards.

For more than 3 alternatives: Majority dynamic does not preserve existence of Condorcet consensus.

Preserving Condorcet Consensus

Preserving Consensus: For every profile, if there exists consensus initially, then for every update order, there will be consensus afterwards.

For more than 3 alternatives: Majority dynamic does not preserve existence of Condorcet consensus.

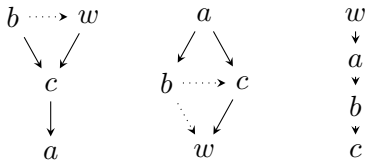


w is the Condorcet winner

Preserving Condorcet Consensus

Preserving Consensus: For every profile, if there exists consensus initially, then for every update order, there will be consensus afterwards.

For more than 3 alternatives: Majority dynamic does not preserve existence of Condorcet consensus.

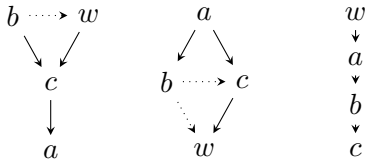


Updating on bc and bw

Preserving Condorcet Consensus

Preserving Consensus: For every profile, if there exists consensus initially, then for every update order, there will be consensus afterwards.

For more than 3 alternatives: Majority dynamic does not preserve existence of Condorcet consensus.

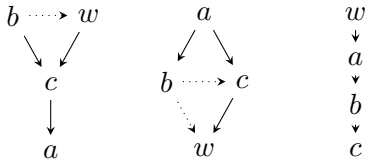


No Condorcet winner

Preserving Condorcet Consensus

Preserving Consensus: For every profile, if there exists consensus initially, then for every update order, there will be consensus afterwards.

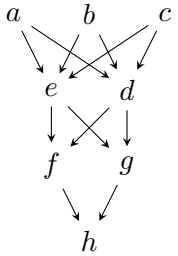
For more than 3 alternatives: Majority dynamic does not preserve existence of Condorcet consensus.



For 3 alternatives and less: Majority dynamic preserves existence of but not identity.

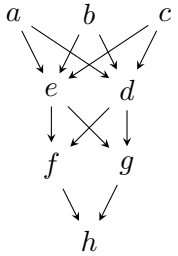
Strict Weak Orders

Strict Weak Orders: alternatives ranked in different levels, incomparabilities within levels



Strict Weak Orders

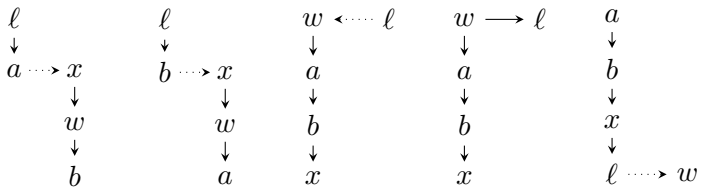
Strict Weak Orders: alternatives ranked in different levels, incomparabilities within levels



With profiles of strict weak orders, the majority dynamic is preserving Condorcet consensus identify.

Quality of the Consensus

A Condorcet loser can be turned into a Condorcet winner.



↪ Condorcet consensus is preserved (w initially and ℓ eventually) but the consensual alternative at the end used to be a Condorcet looser.

From Preservation to Control

So far we focused on preserving consensus, i.e., universal guarantees that the majority dynamic does not harm consensus.

From Preservation to Control

So far we focused on preserving consensus, i.e., universal guarantees that the majority dynamic does not harm consensus.

What's next? Exploring what the decision maker can achieve by selecting a specific update order.

3. Controlling Condorcet Consensus

Positive and Negative Control

Positive Control: The majority dynamics enables positive control if for all profile *with* initial consensus, there exists an update order preserving the consensus.

Positive and Negative Control

Positive Control: The majority dynamics enables positive control if for all profile *with* initial consensus, there exists an update order preserving the consensus.

↪ The decision maker can control the update order to preserve consensus.

Positive and Negative Control

Positive Control: The majority dynamics enables positive control if for all profile *with* initial consensus, there exists an update order preserving the consensus.

↪ The decision maker can control the update order to preserve consensus.

Negative Control: The majority dynamics enables negative control if for all profile *without* initial consensus:

- there exists an update order preserving the *absence* consensus; or,
- two *distinct* consensual alternatives can be reached for different update orders.

Positive and Negative Control

Positive Control: The majority dynamics enables positive control if for all profile *with* initial consensus, there exists an update order preserving the consensus.

↪ The decision maker can control the update order to preserve consensus.

Negative Control: The majority dynamics enables negative control if for all profile *without* initial consensus:

- there exists an update order preserving the *absence* consensus; or,
- two *distinct* consensual alternatives can be reached for different update orders.

↪ The decision maker can control the update order to prevent consensus from happening.

Controlling Condorcet Consensus

Positive Control: The majority dynamics *enables* positive Condorcet consensus control.

Controlling Condorcet Consensus

Positive Control: The majority dynamics *enables* positive Condorcet consensus control.

↳ For a profile with a as initial Condorcet consensus, update according to $ab, ac, ad, ae \ldots$

Controlling Condorcet Consensus

Positive Control: The majority dynamics *enables* positive Condorcet consensus control.

↳ For a profile with a as initial Condorcet consensus, update according to $ab, ac, ad, ae \ldots$

Negative Control: The majority dynamics *enables* negative Condorcet consensus control.

Controlling Condorcet Consensus

Positive Control: The majority dynamics *enables* positive Condorcet consensus control.

↪ For a profile with a as initial Condorcet consensus, update according to $ab, ac, ad, ae \ldots$

Negative Control: The majority dynamics *enables* negative Condorcet consensus control.

↪ We can either easily maintain the absence of Condorcet consensus, or generate two distinct ones for two different update orders.

4. Other Consensus Notions

Our Results

	Preserving consensus	Positive control	Negative control
Condorcet	✗ (✓)	✓	✓

Our Results

	Preserving consensus	Positive control	Negative control
Condorcet	✗ (✓)	✓	✓
Plurality Undominated	✗	✗	✗

Our Results

	Preserving consensus	Positive control	Negative control
Condorcet	✗ (✓)	✓	✓
Plurality Undominated	✗	✗	✗
Plurality Dominant	✗	✗	✗

Our Results

	Preserving consensus	Positive control	Negative control
Condorcet	✗ (✓)	✓	✓
Plurality Undominated	✗	✗	✗
Plurality Dominant	✗	✗	✗
Majority Undominated	✗	✓	✗

Our Results

	Preserving consensus	Positive control	Negative control
Condorcet	✗ (✓)	✓	✓
Plurality Undominated	✗	✗	✗
Plurality Dominant	✗	✗	✗
Majority Undominated	✗	✓	✗
Majority Dominant	✓	✓	✗

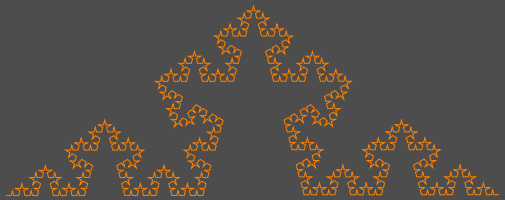
Our Results

	Preserving consensus	Positive control	Negative control
Condorcet	✗ (✓)	✓	✓
Plurality Undominated	✗	✗	✗
Plurality Dominant	✗	✗	✗
Majority Undominated	✗	✓	✗
Majority Dominant	✓	✓	✗
Unanimity Undominated	✗ (✓)	✓	✓

Our Results

	Preserving consensus	Positive control	Negative control
Condorcet	✗ (✓)	✓	✓
Plurality Undominated	✗	✗	✗
Plurality Dominant	✗	✗	✗
Majority Undominated	✗	✓	✗
Majority Dominant	✓	✓	✗
Unanimity Undominated	✗ (✓)	✓	✓
Unanimity Dominant	✓	✓	✗

5. Conclusion



Wrapping Up

What have we done? Studied the majority dynamic and the effects it can have on consensus for several consensus notions.

Wrapping Up

What have we done? Studied the majority dynamic and the effects it can have on consensus for several consensus notions.

What has not been presented? An experimental study to complement the above.

Wrapping Up

What have we done? Studied the majority dynamic and the effects it can have on consensus for several consensus notions.

What has not been presented? An experimental study to complement the above.

What can you do? Several ideas:

- Computational complexity of control problems (selecting the update order to achieve some goal)
- Computational complexity of good update orders (minimising number of updates, etc...)
- Guarantees about distance to consensus when it is not achieve
- And so many others...

Sirin

Simon

Zoi

Come and see our poster yesterday!