Beyond Multiwinner Voting: Participatory Budgeting

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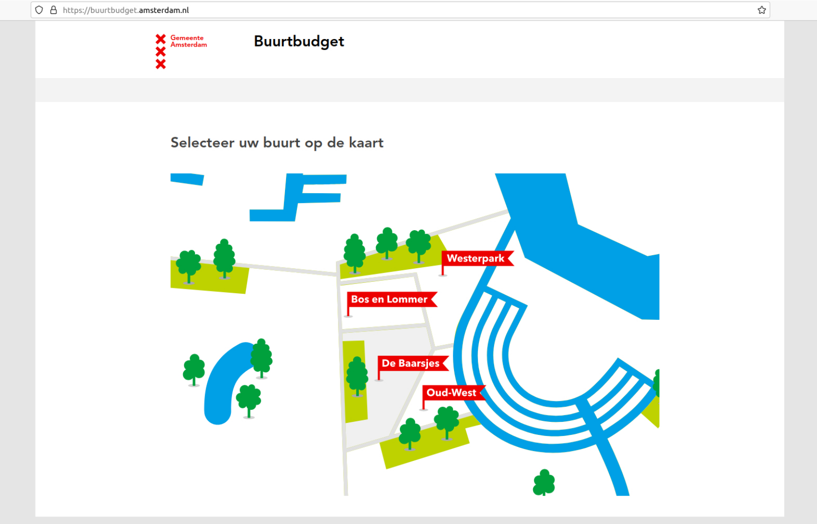
What is Participatory Budgeting

- Participatory Budgeting is a relatively new democratic process, in which citizens can decide how (parts of) their city's budget is spend.
- Invented in 1989 by the worker's party in Porto Alegre, Brazil.
- Today, it is used in more than 1500 cities, including (parts of) Amsterdam.

Participatory Budgeting in Amsterdam

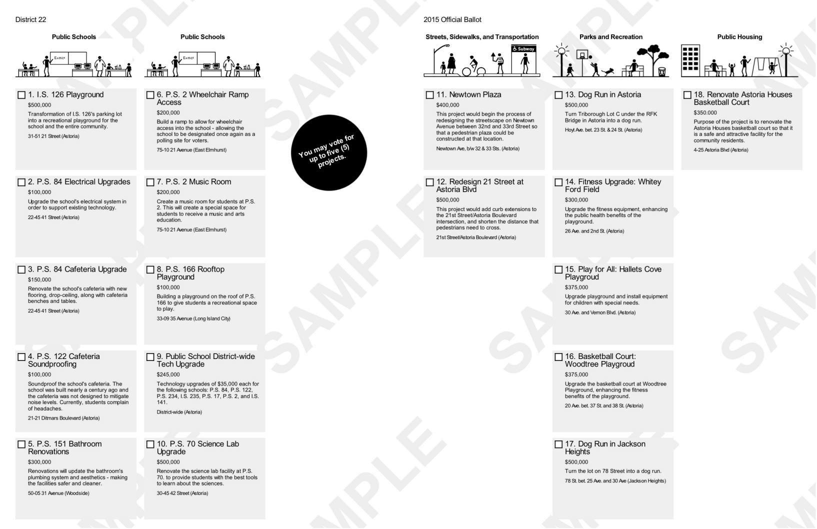


Website: `https://buurtbudget.amsterdam.nl/`

What is Participatory Budgeting

- Participatory Budgeting is usually a two stage process:
  - First, citizens propose projects.
  - Secondly, the citizens vote on which projects should be implemented.
- In recent years, researchers from the area of COMSOC started studying voting rules for the second stage more closely.

What is (Approval-Based) Participatory Budgeting



District 22

2015 Official Ballot

You may vote for up to five (5) projects.

Public Schools

- ☐ 1. I.S. 126 Playground
  $500,000
  Transformation of I.S. 126's parking lot into a recreational playground for the school and the entire community.
  31-51 21 Street (Astoria)
- ☐ 2. P.S. 84 Electrical Upgrades
  $100,000
  Upgrade the school's electrical system in order to support existing technology.
  22-45 41 Street (Astoria)
- ☐ 3. P.S. 84 Cafeteria Upgrade
  $150,000
  Renovate the school's cafeteria with new flooring, drop-ceiling, along with cafeteria benches and tables.
  22-45 41 Street (Astoria)
- ☐ 4. P.S. 122 Cafeteria Soundproofing
  $100,000
  Soundproof the school's cafeteria. The school was built nearly a century ago and the cafeteria was not designed to mitigate noise levels. Currently, students complain of headaches.
  21-21 Ditmars Boulevard (Astoria)
- ☐ 5. P.S. 151 Bathroom Renovations
  $300,000
  Renovations will update the bathroom's plumbing system and aesthetics - making the facilities safer and cleaner.
  50-05 31 Avenue (Woodside)

Public Schools

- ☐ 6. P.S. 2 Wheelchair Ramp Access
  $200,000
  Build a ramp to allow for wheelchair access into the school - allowing the school to be designated once again as a polling site for voters.
  75-10 21 Avenue (East Elmhurst)
- ☐ 7. P.S. 2 Music Room
  $200,000
  Create a music room for students at P.S. 2. This will create a special space for students to receive a music and arts education.
  75-10 21 Avenue (East Elmhurst)
- ☐ 8. P.S. 166 Rooftop Playground
  $100,000
  Building a playground on the roof of P.S. 166 to give students a recreational space to play.
  33-09 35 Avenue (Long Island City)
- ☐ 9. Public School District-wide Tech Upgrade
  $245,000
  Technology upgrades of $35,000 each for the following schools: P.S. 84, P.S. 122, P.S. 234, I.S. 235, P.S. 17, P.S. 2, and I.S. 141.
  District-wide (Astoria)
- ☐ 10. P.S. 70 Science Lab Upgrade
  $500,000
  Renovate the science lab facility at P.S. 70. to provide students with the best tools to learn about the sciences.
  30-45 42 Street (Astoria)

Streets, Sidewalks, and Transportation

- ☐ 11. Newtown Plaza
  $400,000
  This project would begin the process of redesigning the streetscape on Newtown Avenue between 32nd and 33rd Street so that a pedestrian plaza could be constructed at that location.
  Newtown Ave, b/w 32 & 33 Sts. (Astoria)
- ☐ 12. Redesign 21 Street at Astoria Blvd
  $500,000
  This project would add curb extensions to the 21st Street/Astoria Boulevard intersection, and shorten the distance that pedestrians need to cross.
  21st Street/Astoria Boulevard (Astoria)

Parks and Recreation

- ☐ 13. Dog Run in Astoria
  $500,000
  Turn Triborough Lot C under the RFK Bridge in Astoria into a dog run.
  Hoyt Ave. bet. 23 St. & 24 St. (Astoria)
- ☐ 14. Fitness Upgrade: Whitey Ford Field
  $300,000
  Upgrade the fitness equipment, enhancing the public health benefits of the playground.
  26 Ave. and 2nd St. (Astoria)
- ☐ 15. Play for All: Hallets Cove Playgroud
  $375,000
  Upgrade playground and install equipment for children with special needs.
  30 Ave. and Vernon Blvd. (Astoria)
- ☐ 16. Basketball Court: Woodtree Playground
  $375,000
  Upgrade the basketball court at Woodtree Playground, enhancing the fitness benefits of the playground.
  20 Ave. bet. 37 St. and 38 St. (Astoria)
- ☐ 17. Dog Run in Jackson Heights
  $500,000
  Turn the lot on 78 Street into a dog run.
  78 St. bet. 25 Ave. and 30 Ave (Jackson Heights)

Public Housing

- ☐ 18. Renovate Astoria Houses Basketball Court
  $350.000
  Purpose of the project is to renovate the Astoria Houses basketball court so that it is a safe and attractive facility for the community residents.
  4-25 Astoria Blvd (Astoria)

A formal model for Participatory Budgeting

We consider approval based Participatory Budgeting:

- A set of agents N, a set of projects P and a budget limit b.
- For every agent i, the set of projects $A_i \subseteq P$ that i approves.
- A cost function $c : P \to \mathbb{R}^+$.

We call a bundle of projects $W \subseteq P \dots$

... feasible if

$$\sum_{p \in W} c(p) \leq b.$$

... exhaustive if for all $p^* \in P \setminus W$ we have

$$c(p^*) + \sum_{p \in W} c(p) > b.$$

PB vs. approval-based multiwinner voting

- PB instances in which all projects have the same cost are equivalent to approval based multiwinner voting. We call this the unit-cost case.
- When we talk about the unit-cost case, we assume w.l.o.g. that all projects have cost 1.
- We can consider PB as "weighted" multiwinner voting.
- Observe: In multiwinner voting exhaustiveness is essentially always required!

PB rules

- A PB rule R is a function that maps a PB instance $(\boldsymbol{A}, c, b)$ to a feasible bundle $R(\boldsymbol{A}, c, b)$.
- We say a PB rule is exhaustive if its output is always exhaustive.

An example PB instance

	p_1	p_2	p_3	p_4	p_5
$c(\cdot)$	8	8	5	5	2
1	✓	✓	✓	×	×
2	✓	✓	✓	×	×
3	✓	✓	✓	×	×
4	✓	×	×	✓	×
5	×	✓	×	✓	×
6	✓	×	×	×	✓
7	×	×	×	×	✓
# of app.	5	4	3	2	2

- $b = 14$
- Most common PB rule is Greedy approval:
  - Take most approved project first
  - Remove projects which are no longer affordable
  - Take most approved project among the remaining projects
  - Repeat until no further project is affordable.
- Winning bundle in example: $\{p_1, p_3\}$

A problematic PB instance

	p_1	p_2	p_3	p_4	p_5
$c(\cdot)$	8	8	5	5	2
1	✓	✓	✓	×	×
2	✓	✓	✓	×	×
3	✓	✓	✓	×	×
4	×	×	×	✓	×
5	×	×	×	✓	×
6	×	×	×	×	✓
7	×	×	×	×	✓
# of app.	3	3	3	2	2

- $b = 14$
- Most common PB rule is Greedy approval.
- Winning bundle in example: $\{p_1, p_3\}$ or $\{p_2, p_3\}$.
- $1, 2, 3$ are not even a majority, but nevertheless only their projects are funded!

Justified representation in PB

- Fairer PB rules are needed!
- Natural idea: Lift proportionality axioms and proportional rules from multiwinner voting to PB.
- In this lecture, we will try to lift the axiom EJR and the rule MES to the PB setting.
- In doing so, we will encounter three problems:
    - Cohesive groups need to be redefined.
    - We need to reason about the satisfaction of an agent.
    - Problems that were easy in MWV become intractable.

Reminder: Cohesive Groups in MWV

ℓ-Cohesive Groups

A group of agents $N' \subseteq N$ is called ℓ-cohesive if $|N'| \geq \ell n/k$ and $|\bigcap_{i \in N'} A_i| \geq \ell$.

A problematic PB instance

	p_1	p_2	p_3	p_4	p_5
$c(\cdot)$	8	8	5	5	2
1	✓	✓	✓	×	×
2	✓	✓	✓	×	×
3	✓	✓	✓	×	×
4	×	×	×	✓	×
5	×	×	×	✓	×
6	×	×	×	×	✓
7	×	×	×	×	✓
# of app.	3	3	3	2	2

- $b = 14$
- 4 and 5 represent $^2/_7$ of the agents and agree on projects worth more than $^2/_7$ of the budget.
- They have no project that they can afford from $^2/_7$ of the budget.

T-Cohesive Groups

ℓ-Cohesive Groups

A group of agents $N' \subseteq N$ is called ℓ-cohesive if $|N'| \geq \ell n/k$ and $|\bigcap_{i \in N'} A_i| \geq \ell$.

T-Cohesive Groups

Let $T \subseteq P$ be a set of projects. Then a group of agents $N' \subseteq N$ is called T-cohesive if $c(T) \leq b \cdot |N'|/n$ and $T \subseteq \bigcap_{i \in N'} A_i$.

A problematic PB instance

	p_1	p_2	p_3	p_4	p_5
$c(\cdot)$	8	8	5	5	2
1	✓	✓	✓	×	×
2	✓	✓	✓	×	×
3	✓	✓	✓	×	×
4	×	×	×	✓	×
5	×	×	×	✓	×
6	×	×	×	×	✓
7	×	×	×	×	✓
# of app.	3	3	3	2	2

- $b = 14$
- 6 and 7 are $\{p_5\}$-cohesive.
- 4 and 5 are not $\{p_4\}$-cohesive as $c(\{p_4\} > 14 \cdot 2/7 = 4$

ℓ-cohesiveness vs. T-cohesiveness

Proposition

Let (A, b, c) be a unit-cost PB instance. Then a group of agents N' is ℓ-cohesive if and only if there is a set $T \subseteq P$ with $c(T) = \ell$ for which N' is T-cohesive.

- Assume N' is ℓ-cohesive.
- Then $|\bigcap_{i \in N'} A_i| \geq \ell$.
- Let T be ℓ arbitrary projects from $|\bigcap_{i \in N'} A_i|$.
- Then $c(T) = \ell \leq b \cdot {|N'|}/{n}$ and $T \subseteq \bigcap_{i \in N'} A_i$.
- Hence N' is T-cohesive.

- Assume N' is T-cohesive and $c(T) = \ell$.
- Then $\ell = c(T) \leq b \cdot {|N'|}/{n}$ and $T \subseteq \bigcap_{i \in N'} A_i$.
- Hence $|\bigcap_{i \in N'} A_i| \geq c(T) = |T| = \ell$ and $|N'| \geq \ell {n}/{b}$.
- Hence N' is ℓ-cohesive.

Reminder: EJR in multiwinner voting

EJR

A committee $W \subseteq C$ of size k satisfies EJR if for every $\ell \in \{1, \ldots, k\}$ and every ℓ-cohesive group N', there exists a $i \in N'$ such that $|A_i \cap W| \geq \ell$.

Satisfaction of an agent

The three most common options to model an agents satisfaction:

- Count the number of approved projects: $\mu_i^{\#}(W) = |A_i \cap W|$.
- Cost of approved projects: $\mu_i^c(W) = c(A_i \cap W)$.
- Assume that agents provide an (additive) cardinal utility $\mu_i(p)$ for each project p: $\mu_i(W) = \sum_{p \in W} \mu_i(p)$.

Reasons against counting number of projects

Count the number of approved projects: $\mu_i^{\#}(W) = |A_i \cap W|$.

- Difference in cost between projects can be huge.
- Particularly problematic when we are concerned with fairness: In example below $\{p_1, p_2\}$ and $\{p_1, p_3\}$ would be considered equally fair.

	p_1	p_2	p_3
$c(\cdot)$.	50	50	1
1	✓	×	×
2	×	✓	✓

Reasons against taking cost of projects

Cost of approved projects: $\mu_i^c(W) = c(A_i \cap W)$.

- Implementing the same project in different locations might produce different cost. No reason to assume that this leads to a change in satisfaction.
- Higher cost might lead to "diminishing marginal returns".
- Rules based on this satisfaction principle often don't satisfy "discount monotonicity", i.e., making a winning project cheaper might lead to the project not winning anymore.

Observation: Greedy approval is implicitly based on this assumption.

Reasons against eliciting cardinal utilities

Assume that agents provide an (additive) cardinal utility $\mu_i(p)$ for each project p: $\mu_i(W) = \sum_{p \in W} \mu_i(p)$.

- Places a high cognitive load on the agents.
- Interpersonal comparison between different cardinal utilities is problematic.
- Additivity is still a very strong assumption.

Possible solutions (Work in progress)

- Define a general theory of "approval-based" satisfaction functions. (Work in progress together with Markus Brill, Martin Lackner, Stefan Forster and Jannik Peters).
- Define fairness without using the concept of satisfaction. (Work in progress together with Simon Rey, Martin Lackner and Ulle Endriss; possible topic for a student presentation)

For this talk we focus on $\mu_i^{\#}$ and μ_i^c. Cardinal utilities are a possible topic for a student presentation.

EJR-$\mu_i^{\#}$

EJR

A committee $W \subseteq C$ of size k satisfies EJR if for every $\ell \in \{1, \ldots, k\}$ and every ℓ-cohesive group N', there exists a $i \in N'$ such that $|A_i \cap W| \geq \ell$.

EJR-$\mu_i^{\#}$

A bundle $W \subseteq P$ satisfies EJR-$\mu_i^{\#}$ if for every $T \subseteq P$ and every T-cohesive group N', there exists a $i \in N'$ such that $|A_i \cap W| \geq |T|$.

EJR and EJR-$\mu_i^{\#}$ are equivalent in the unit-cost case. This follows directly from the equivalence of ℓ- and T-cohesiveness in the unit-cost case.

Satisfying EJR-$\mu_i^\#$

We can naturally adapt MES to the PB setting:

- Every agent starts with a budget of b/n.
- The price of project p is $c(p)$.
- A project p is α-affordable in round t if

$$\sum_{i \in N; p \in A_i} \min \alpha, b_i(t) \geq c(p)$$

- In every round, the project that is α-affordable for the smallest α is selected.

Theorem

MES satisfies EJR-$\mu_i^\#$.

EJR-μ_i^c

EJR-$\mu_i^{\#}$

A bundle $W \subseteq P$ satisfies EJR-$\mu_i^{\#}$ if for every $T \subseteq P$ and every T-cohesive group N', there exists a $i \in N'$ such that $|A_i \cap W| \geq |T|$.

EJR-μ_i^c

A bundle $W \subseteq P$ satisfies EJR-μ_i^c if for every $T \subseteq P$ and every T-cohesive group N', there exists a $i \in N'$ such that $c(A_i \cap W) \geq c(T)$.

EJR, EJR-$\mu_i^{\#}$ and EJR-μ_i^c are equivalent in the unit-cost case, as $|S| = c(S)$ for all $S \subseteq P$ in unit-cost instances.

EJR-μ_i^c vs. EJR-$\mu_i^\#$

In general, EJR-μ_i^c and EJR-$\mu_i^\#$ are incompatible. Consider the following instance with $b = 5$.

	p_1	p_2	p_3	p_4
$c(\cdot)$	5	1	1	1
1	✓	✓	✓	✓

Then, only $\{p_1\}$ satisfies EJR-μ_i^c while only $\{p_2, p_3, p_4\}$ satisfies EJR-$\mu_i^\#$

Complexity woes

Theorem

If there is a polynomial time computable PB rule that satisfies EJR-μ_i^c, then P = NP.

Complexity woes: Proof

- Consider the problem SUBSET SUM: Given a set of positive integers S and an integer t, is there a subset $S' \subseteq S$ such that $\sum_{s \in S'} s = t$?
- SUBSET SUM is known to be NP-complete.
- Given a SUBSET SUM instance (S, t), construct a PB instance with
  - 1 agent,
  - one project p_s for each $s \in S$
  - $c(p_s) = s$ and $b = t$.
- Use PB rule to find a bundle W that satisfies EJR-μ_i^c in polynomial time.
- (S, t) is a positive instance if and only if $c(W) = t$.

Satisfiability of EJR-μ_i^c

Greedy Cohesive for Cost

- Initialize W and N^* as empty sets.
- While exists $N' \subseteq N \setminus N^*$ and $T \subseteq P \setminus W$, s.t. N' is T-cohesive:
  - Select $N'' \subseteq N \setminus N^*$ and $T' \subseteq P \setminus W$ with maximal cost such that N'' is T'-cohesive.
  - Add T' to W and N'' to N^*.

Theorem

Greedy Cohesive for Cost satisfies EJR-μ_i^c.

Observe: Greedy Cohesive for Cost requires exponential runtime.

Satisfiability of EJR-μ_i^c: Proof Idea

Theorem

Greedy Cohesive for Cost satisfies EJR-μ_i^c.

Proof idea

- Let N' be a T-cohesive group.
- First case: N' was "picked" by the algorithm in some round. Then EJR-μ_i^c is satisfied by construction.
- Second case: N' was never picked.
- Then, either all projects in T have been picked...
- ...or at least one agent $i \in N'$ has been part of another group that has been picked.
- By the maximality criterion for picking groups, this implies $c(A_i \cap W) \geq c(T)$.

Relaxing EJR-μ_i^c

EJR-μ_i^c

A bundle $W \subseteq P$ satisfies EJR-μ_i^c if for every $T \subseteq P$ and every T-cohesive group N', there exists a $i \in N'$ such that $c(A_i \cap W) \geq c(T)$.

EJR-μ_i^c up to one

A bundle $W \subseteq P$ satisfies EJR-μ_i^c up to one if for every $T \subseteq P$ and every T-cohesive group N', there exists a $i \in N'$ such that either $c(A_i \cap W) \geq c(T)$ or there exists a $p \in P \setminus W$ such that $c(A_i \cap (W \cup \{p\})) > c(T)$.

In the unit cost case, we have $c(A_i \cap W) \geq c(T)$ iff $c(A_i \cap (W \cup \{p\})) > c(T)$. Therefore, EJR, EJR-$\mu_i^{\#}$, EJR-$\mu_i^c$ and EJR-μ_i^c up to one are equivalent in the unit-cost case.

Satisfying EJR-μ_i^c up to one

We adapt MES to take cost into account. MES[μ_i^c] is defined as

- Every agent starts with a budget of b/n.
- The price of project p is $c(p)$.
- A project p is α-affordable in round t if

$$\sum_{i \in N; p \in A_i} \min \alpha \cdot c(p), b_i(t) \geq c(p)$$

- In every round, the project that is α-affordable for the smallest α is selected.

Theorem

MES[μ_i^c] satisfies EJR-μ_i^c up to one.

Satisfying EJR-μ_i^c up to one: Proof idea

- First observe that $1/\alpha$ is the satisfaction per money of an agent that pays "full price".
- Let N' be a T-cohesive group.
- Assume round k is the first round after which an agent i in N' could not pay "in full" for a project p in T. Let W^* be the projects selected in rounds 1 to k.
- To get $A_i \cap (W^* \cup \{p\})$ agent i would need to spend more than b/n.
- Before round k, all agents in N' could pay for all projects in T in full. Hence projects in W^* give at least as much satisfaction per money as agents in N' get by buying the projects in T by themselves.
- It follows that $c(A_i \cap (W^* \cup \{p\})) > c(T)$.

Unpublished bonus result

EJR-μ_i^c up to any

A bundle $W \subseteq P$ satisfies EJR-μ_i^c up to any if for every $T \subseteq P$ and every T-cohesive group N', there exists a $i \in N'$ such that either $c(A_i \cap W) \geq c(T)$ or for every $p \in P \setminus W$ we have $c(A_i \cap (W \cup \{p\})) > c(T)$.

Theorem

MES[μ_i^c] satisfies EJR-μ_i^c up to any.

Summary

- PB is a generalization of mulitwinner voting.
- We can lift proportionality axioms like EJR from multiwinner voting to PB.
- In order to talk about proportionality, we have to fix a satisfaction function for the agents.
- If we assume $\mu_i^{\#}$ describes agents' satisfaction, then we can achieve EJR-$\mu_i^{\#}$ in polynomial time using MES.
- If we assume μ_i^{c} describes agents' satisfaction, then we can only achieve EJR-μ_i^{c} up to any in polynomial time using MES[μ_i^{c}].