A Review of the Computational Social Choice Literature on Participatory Budgeting

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Introduction

Participatory Budgeting (PB) is a *decision-making* process where *citizen* deliberate and negotiate over the *distribution* of *public resources*.

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[1] Shah *Participatory budgeting* (2007)

Participatory budgeting step by step

1. The municipality is divided into regions to facilitate meetings.
2. Each area is allocated a given share of the budget.
3. Citizen debate and negotiate to submit project propositions.
4. The city council together with experts decide on a shortlist of the propositions.
5. Citizen, or representatives, vote to select the projects to be funded.

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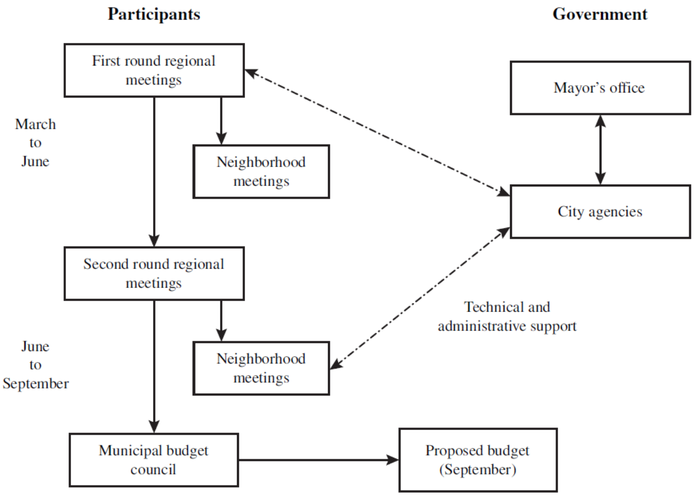
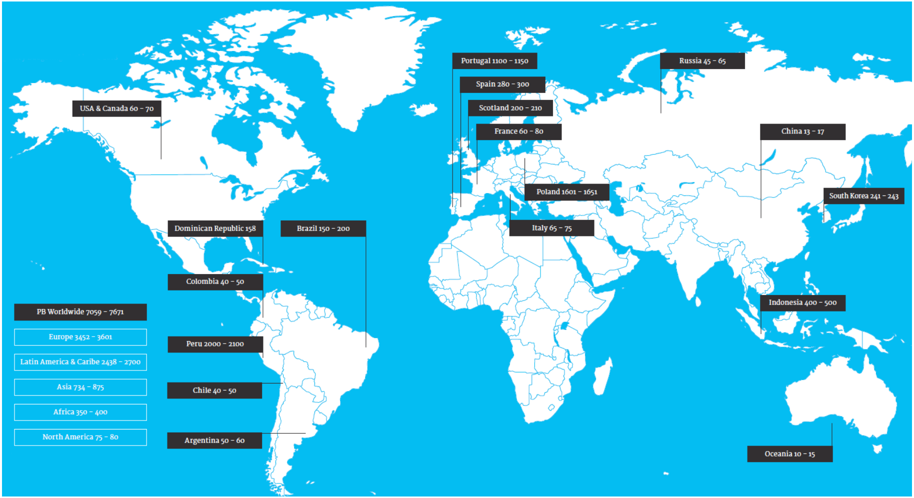
➥ Citizen are asked *twice* their input during the process.

Participatory budgeting step by step



Source: Wampler 2000.

Participatory budgeting map



[2] Dias *Hope for democracy: 30 years of participatory budgeting* (2018)

Participatory budgeting in Amsterdam

Gemeente Amsterdam

Buurtbudget Slotermeer Noordoost

> English > Türkçe > العربية

Home Stemmen

Participatory Budgeting for Slotermeer Northeast

You know what is best for your neighborhood. That is why this past spring we started with the Buurtbudget (participatory budgeting) for Slotermeer Northeast, where you get to decide how we spend €500.000. Now you can vote for your favorite plans and decide which plans will be implemented. You can vote (in Dutch) from October 7th until November 4th 2019 on this website.

How does it work?

1. Click on 'stemmen'
2. Choose your favorite plans
3. Enter the personal voting code provided in the letter you received from the municipality
4. The plans with the most votes will be implemented

Help with voting

The website is in Dutch, but we want everyone to be able to participate!

You can try to translate the website using Google Translate. It's not perfect but a pretty decent translation.

You can call or WhatsApp us directly (0639004343) if you need assistance with voting. You can also come by our office hours. We are happy to help you with the voting process.

Participatory budgeting in Amsterdam

Plein '40-'45 in 2020 het schoonste pl...

Veilige, groene en prettige buurt |

> Lees meer

€ 40.000 | 639 stemmen

Maak een groene tuinkade van de Ja...

Veilige, groene en prettige buurt |

> Lees meer

€ 40.000 | 529 stemmen

Schaakbord op Plein '40-'45

Veilige, groene en prettige buurt |

> Lees meer

€ 1.700 | 521 stemmen

Marktafval Plein '40-'45 scheiden en ...

Veilige, groene en prettige buurt |

> Lees meer

€ 90.000 | 510 stemmen

Bewoners helpen met financiële probl...

Samen dingen doen |

> Lees meer

€ 5.000 | 495 stemmen

Bloemrijke klimaattuin bij de Burgem...

Veilige, groene en prettige buurt |

> Lees meer

€ 50.000 | 488 stemmen

1. The Model

Basic components

We consider the model presented by [3]. We consider:

- A *set of resources* $\mathcal{R}$, there are d of them,
- A *budget limit* for each resource $\mathbf{B} = \langle B_r \rangle_{r \in \mathcal{R}}$,
- A *set of project* $\mathcal{P}$ of size m,
- For each project $p \in \mathcal{P}$, a set of *completion degree* χ_p with $0 \in \chi_p$,
- For each project $p \in \mathcal{P}$, a *cost function* $c_p : \chi_p \to \mathbb{R}^d$,
- A *set of agent* $\mathcal{N}$ who express *preferences* over the project.

[3] Aziz and Shah "Participatory Budgeting: Models and Approaches" (2019)

Budget allocation

A *budget allocation* $\pi = \langle \pi_p \rangle_{p \in \mathcal{P}}$ is a tuple specifying for each project the completion degree selected.

> **Definition: Feasible budge allocation**
>
> A budget allocation π is said to be feasible if and only if:
>
> $$\sum_{p \in \mathcal{P}} c_p(\pi_p) \leq \mathbf{B}.$$

Taxonomy of the participatory budgeting problems

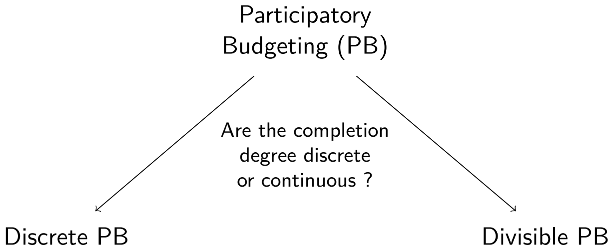
Participatory
Budgeting (PB)

Taxonomy of the participatory budgeting problems

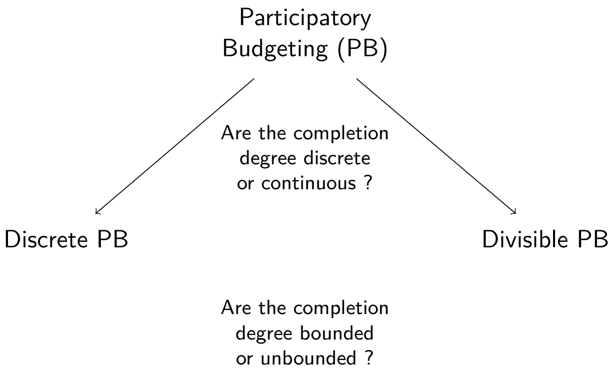
Participatory
Budgeting (PB)

Are the completion
degree discrete
or continuous ?

Taxonomy of the participatory budgeting problems



Taxonomy of the participatory budgeting problems



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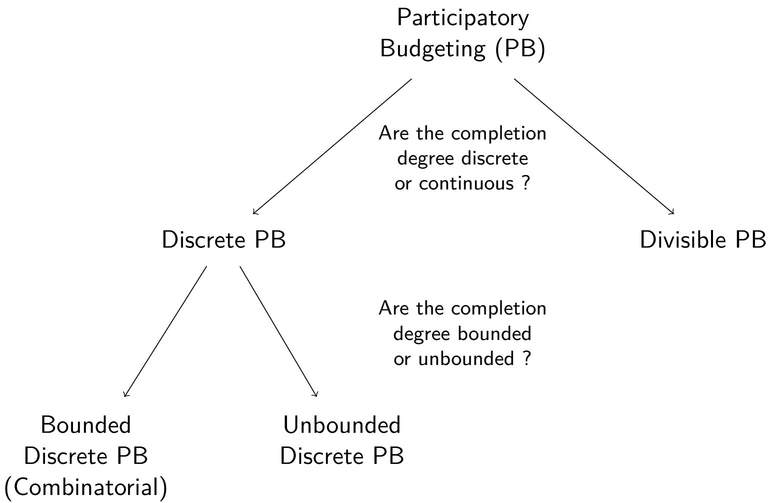

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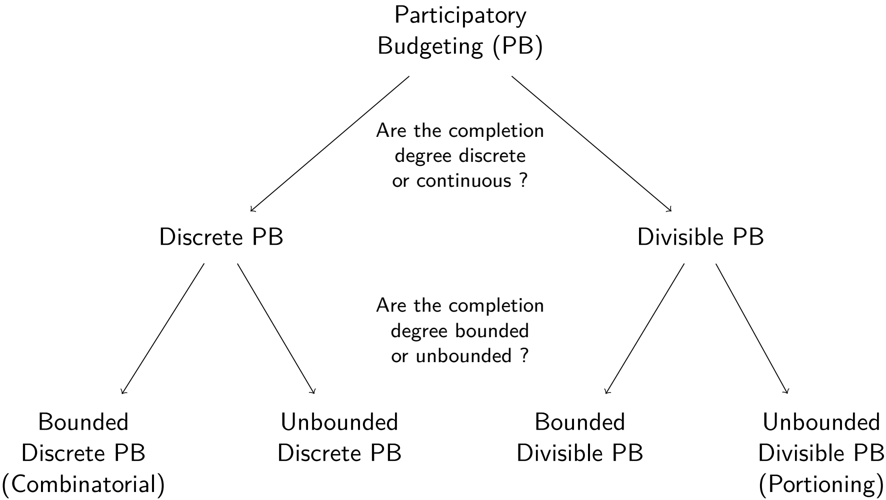

2. Divisible Participatory Budgeting

Divisible Participatory Budgeting

Welfare maximization

Welfare maximization

With *bounded divisible PB*: sorting the projects by *value-for-money* and *greedily* fund them is enough to maximize *the utilitarian welfare* [4].

[4] Goel, Krishnaswamy, Sakshuwong, and Aitamurto "Knapsack voting: Voting mechanisms for participatory budgeting" (2019)
[5] Garg, Kamble, Goel, Marn, and Munagala "Iterative local voting for collective decision-making in continuous spaces" (2019)

Welfare maximization

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For the public decision making problem over a *multi-dimensional continuous space* with *utility based on the norm* l_p, the *Iterative local voting* class of algorithms [5] converges to *welfare maxima*.

[4] Goel, Krishnaswamy, Sakshuwong, and Aitamurto "Knapsack voting: Voting mechanisms for participatory budgeting" (2019)
[5] Garg, Kamble, Goel, Marn, and Munagala "Iterative local voting for collective decision-making in continuous spaces" (2019)

Iterative local voting

Input: An initial solution x_0, a tolerance ϵ, an integer N, an initial radius r_0, a terminaison time T and a norm q.
Output: A solution x
Set $t = 1$.
while $t \leq T$ **do**
  Let a_t be a random agent.
  Set $r_t = r_0 / t$.
  Elicit the value: $x_t = \arg\max_{x \text{ a solution within } r_t \text{ from } x_{t-1}} u_{a_t}(x)$.
  if all previous N solutions are within ϵ from the agents' top **then**
    return x_t.
return x_T.

Divisible Participatory Budgeting

Fairness and incentives

The core of a divisible PB problem

Definition: Core

A budget allocation π is in the core if there is no subset of agents N such that by using $|N|/n$ of the budget they can find a budget allocation Pareto-dominating π.

[6] Fain, Goel, and Munagala "The core of the participatory budgeting problem" (2016)

The core of a divisible PB problem

DEFINITION: CORE

A budget allocation π is in the core if there is no subset of agents N such that by using $|N|/n$ of the budget they can find a budget allocation Pareto-dominating π.

With *scalar separable utility functions*, a budget allocation in the core can *always* be computed in *polynomial time* [6].

Their algorithm computes a Lindahl equilibrium (an equilibrium with different prices) via a *convex program* which is show to *always be in the core*.

[6] Fain, Goel, and Munagala "The core of the participatory budgeting problem" (2016)

Proportionality

DEFINITION:

Assume that the agents are all *single-minded*. A budget allocation π is *proportional* if $\forall p \in P, \pi(p) = \frac{|\{i \in \mathcal{N} | p \in A_i\}|}{n}$.

[7] Freeman, Pennock, Peters, and Vaughan "Truthful Aggregation of Budget Proposals" (2019)

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Some of the phantoms mechanisms [7] are proportional.

- For 2 projects: a *proportional* and *incentive-compatible* mechanism.
- For any number of projects: a *proportional* mechanism and *incentive-compatible* mechanism.
- For any number of projects: a characterization of a subset of *Pareto-optimal* and *incentive-compatible* mechanisms, none of which are *proportional*.

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[7] Freeman, Pennock, Peters, and Vaughan "Truthful Aggregation of Budget Proposals" (2019)

The uniform phantom mechanism

Theorem: Moulins' characterization [8]

For two projects, an anonymous and continuous mechanism $\mathcal{A}$ is incentive compatible if and only if there are $\alpha_0 \geq \alpha_1 \geq \cdots \geq \alpha_n$ in $[0, 1]$ such that for every profile P:

$$\mathcal{A}(P)_1 = med(p_{1,1}, p_{2,1}, \ldots, p_{n,1}, \alpha_0, \ldots, \alpha_n)$$
$$\mathcal{A}(P)_2 = med(p_{1,2}, p_{2,2}, \ldots, p_{n,2}, 1 - \alpha_0, \ldots, 1 - \alpha_n)$$

The uniform phantom mechanism is such that: $\forall k \in [0, 1], \alpha_k = 1 - \frac{k}{n}$.
It is the *unique* anonymous and continuous mechanism that is both *incentive compatible* and *proportional*.

[8] Moulin "On strategy-proofness and single peakedness" (1980)

Divisible Participatory Budgeting

Unbounded completion degree

Portioning for unbounded divisible PB

The unbounded divisible PB is very close to the portioning problem [9, 10].

- For ordinal preferences, [9] introduced a group fairness criteria and showed that some positional scoring rules do satisfy it.
- For dichotomous preferences, [10] investigated some rules to look for proportional fairness guarantees and strategy-proofness.

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[9] Airiau, Aziz, Caragiannis, Kruger, Lang, and Peters "Portioning Using Ordinal Preferences: Fairness and Efficiency" (2019)
[10] Aziz, Bogomolnaia, and Moulin "Fair mixing: the case of dichotomous preferences" (2019)

3. Combinatorial Participatory Budgeting

Combinatorial Participatory Budgeting

Preferences Elicitation

Ballots

- *k-approval*: each agent submit a subset of the projects of size at most k she approves of.

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- *Threshold approval vote*: for a given threshold t, the agents submit the subset of projects giving that much utility.
- *Independent threshold approval vote*: each agent is given an individual threshold t and submits the subset of projects giving that much utility.

Comparing the ballots

Definition: Distortion

The distortion of an elicitation method is the worst case ratio between the optimal social welfare and the achieved one.

[11] Benade, Nath, Procaccia, and Shah "Preference elicitation for participatory budgeting" (2017)
[12] Bhaskar, Dani, and Ghosh "Truthful and near-optimal mechanisms for welfare maximization in multi-winner elections" (2018)
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- The distortion for the *knapsack voting* is exponentially bad.
- The distortion for the *ranking by value* is polynomially bad.
- The distortion for the *threshold approval voting* is logarithmically bad.
- The distortion for the *independent threshold voting* is almost 1.

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[12] Bhaskar, Dani, and Ghosh "Truthful and near-optimal mechanisms for welfare maximization in multi-winner elections" (2018)
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Real world experiments

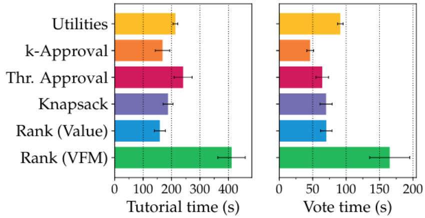

Figure 4: Average time taken (in seconds) to complete the pre-task tutorial and to cast a vote in each input format. Lower is better.

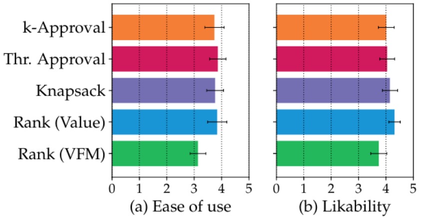

Figure 5: How easy to use each input format is, and how liked its user interface is, based on the subjective reports of the voters on a scale of 0 to 5, 5 being the best.

[14] Benade, Itzhak, Shah, Procaccia, and Gal "Efficiency and usability of participatory budgeting methods" (2018)

Real world experiments

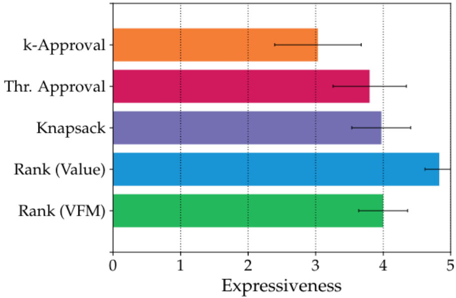

Figure 6: Voters' perceived expressiveness of different input formats. Higher is better.

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Combinatorial Participatory Budgeting

Maximizing the social welfare

Knapsack complexity

Maximizing the utilitarian welfare in combinatorial PB is solving the knapsack problem. This problem is one of the classic *NP-hard problem* [15]. There exist a *pseudo-polynomial algorithm* and a *FPTAS* to solve it [16].

[15] Karp "Reducibility among combinatorial problems" (1972)
[16] Vazirani *Approximation algorithms* (2013)
[17] Fluschnik, Skowron, Triphaus, and Wilker "Fair knapsack" (2019)

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Other knapsacks can be considered [17]:

- *Diverse knapsack*: aggregate atomic utilities with a max function. It is *NP-hard* and weakly NP-hard with single-peaked or single-crossing preferences. There is a *FPT* parametrized by the number of voters.
- *Fair knapsack*: aggregate atomic utilities with the Nash product. It is *NP-hard* to find such knapsack and *W[1]-hard* when parametrized by the number of voters.

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[16] Vazirani *Approximation algorithms* (2013)
[17] Fluschnik, Skowron, Triphaus, and Wilker "Fair knapsack" (2019)

Combinatorial Participatory Budgeting

Monotonicity axioms

Monotonicity axioms

- *Budget monotonicity*: it should not be possible to fund extra project with the remaining budget (also called exhaustiveness [18]).

[19] Faliszewski and Talmon "A framework for approval-based budgeting methods" (2019)
[18] Aziz, Lee, and Talmon "Proportionally representative participatory budgeting: Axioms and algorithms" (2018)

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- *Discount monotonicity*: if a funded project becomes cheaper it should still be funded.
- *Splitting and merging monotonicity*: a funded project can be split into several smaller ones which will all be funded. Multiple funded projects (all approved by the same agents) can be gathered into one project which should be funded.

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Satisfying monotonicity axioms

They consider different satisfaction functions for the agents:

- $|B_v|$: the utility is the number of approved and selected projects.
- $|B_v| > 0$: the utility is 1 if at least one approved project has been selected and 0 otherwise.
- $c(B_v)$: the utility is the cost of the approved and selected projects.

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They consider different selection procedures for the projects:

- *Max rules*: maximize the utilitarian social welfare.
- *Greedy rules*: iteratively selects projects by adding the one with maximum increase in the utilitarian social welfare.
- *Proportional greedy rules*: iteratively selects projects by adding the one with maximum proportional increase in the utilitarian SW.

[19] Faliszewski and Talmon "A framework for approval-based budgeting methods" (2019)

Satisfying monotonicity axioms

	$\lvert B_v \rvert$			$\lvert B_v \rvert > 0$			$c(B_v)$		
	R^m	R^g	R^p	R^m	R^g	R^p	R^m	R^g	R^p
Complexity	P	P	P	NP-h	P	P	w NP-h	P	P

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Complexity	P	P	P	NP-h	P	P	w NP-h	P	P
Budget	✓	✓	✓	✓	✓	✓	✓	✓	✓

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Satisfying monotonicity axioms

	$\lvert B_v \rvert$			$\lvert B_v \rvert > 0$			$c(B_v)$		
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Complexity	P	P	P	NP-h	P	P	w NP-h	P	P
Budget	✓	✓	✓	✓	✓	✓	✓	✓	✓
Discount	✓	✓	✓	✓	✓	✓	✗	✗	✗

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Complexity	P	P	P	NP-h	P	P	w NP-h	P	P
Budget	✓	✓	✓	✓	✓	✓	✓	✓	✓
Discount	✓	✓	✓	✓	✓	✓	✗	✗	✗
Splitting	✓	✓	✓	✓	✓	✓	✓	✗	✓
Merging	✗	✗	✗	✓	✓	✗	✓	✓	✓

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Discount	✓	✓	✓	✓	✓	✓	✗	✗	✗
Splitting	✓	✓	✓	✓	✓	✓	✓	✗	✓
Merging	✗	✗	✗	✓	✓	✗	✓	✓	✓
Limit	✗	✗	✗	✗	✗	✗	✗	✗	✗

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Combinatorial Participatory Budgeting

Representation axioms

Proportional representativeness axioms

The justified representation axiom from multi-winner voting can be adapted to participatory budgeting [18].

<u>Definition</u>: Strong-BJR

A budget allocation π satisfies Strong-BJR if there is no $N \subseteq \mathcal{N}$ such that $|N| \geq \frac{n}{\mathbf{B}}$ such that $c\left(\bigcap_{i \in N} A_i\right) \geq 1$ and $c\left(\pi \cup \bigcup_{i \in N} A_i\right) = 0$.

[18] Aziz, Lee, and Talmon "Proportionally representative participatory budgeting: Axioms and algorithms" (2018)

Proportional representativeness axioms

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Definition: Strong-BPJR

A budget allocation π satisfies Strong-BPJR if for every $l \in [1, \mathbf{B}]$, there is no $N \subseteq \mathcal{N}$ such that $|N| \geq \frac{l * n}{\mathbf{B}}$ such that $c\left(\bigcap_{i \in N} A_i\right) \geq l$ and $c\left(\pi \cup \bigcup_{i \in N} A_i\right) < l$.

[18] Aziz, Lee, and Talmon "Proportionally representative participatory budgeting: Axioms and algorithms" (2018)

Proportional representativeness axioms

	Existence	Complexity of testing	Complexity of computing
Strong-BJR	✗	P	NP-h
Strong-BPJR	✗	co-NP-c	NP-h

[18] Aziz, Lee, and Talmon "Proportionally representative participatory budgeting: Axioms and algorithms" (2018)

Proportional representativeness axioms

Definition: BJR

A budget allocation π satisfies BJR if there is no $N \subseteq \mathcal{N}$ such that $|N| \geq \frac{n}{\mathbf{B}}$ such that $c\left(\bigcap_{i \in N} A_i\right) \geq 1$, $c\left(\pi \cup \bigcup_{i \in N} A_i\right) = 0$ and there is $p \in \bigcap_{i \in N} A_i$ *such that* $c(p) = 1$.

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Definition: BPJR

A budget allocation π satisfies BPJR if for every $l \in [1, \mathbf{B}]$, there is no $N \subseteq \mathcal{N}$ such that $|N| \geq \frac{l * n}{\mathbf{B}}$ such that $c\left(\bigcap_{i \in N} A_i\right) \geq l$ and:

$$c\left(\pi \cup \bigcup_{i \in N} A_i\right) < \max\left\{c(P') \mid P' \subseteq \bigcap_{i \in N} A_i \text{ and } c(P') \leq \frac{|N| * \mathbf{B}}{n}\right\}.$$

Proportional representativeness axioms

	Existence	Complexity of testing	Complexity of computing
BJR	✓	P	P
Strong-BJR	✗	P	NP-h
BPJR	✓	co-NP-c	NP-h
Strong-BPJR	✗	co-NP-c	NP-h

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Proportional representativeness axioms

DEFINITION: LOCAL-BPJR

A budget allocation π satisfies Local-BPJR if for every $l \in [1, \mathbf{B}]$, there is no $N \subseteq \mathcal{N}$ with $|N| \geq \frac{l * n}{\mathbf{B}}$ such that $c\left(\bigcap_{i \in N} A_i\right) \geq l$ and there exists:

$$P \in \arg\max\{c(P') \mid P' \subseteq \bigcap_{i \in N} A_i \text{ and } c(P') \leq l\}$$

with:

$$\left(\pi \cap \bigcup_{i \in N} A_i\right) \subset P.$$

[18] Aziz, Lee, and Talmon "Proportionally representative participatory budgeting: Axioms and algorithms" (2018)

Proportional representativeness axioms

	Existence	Complexity of testing	Complexity of computing
BJR	✓	P	P
Strong-BJR	✗	P	NP-h
Local BPJR	✓	co-NP-c	P
BPJR	✓	co-NP-c	NP-h
Strong-BPJR	✗	co-NP-c	NP-h

[18] Aziz, Lee, and Talmon "Proportionally representative participatory budgeting: Axioms and algorithms" (2018)

Computing proportionally representative budgets

Algorithm 1: Generalized Phragmen's sequential rule

Input: An instance $I = \langle \mathcal{N}, \mathcal{P}, \mathbf{B} \rangle$.

Output: A budget allocation π.

Set $\pi = \emptyset$.

while $P' = \{p \notin \pi \mid c(\pi) + c(p) \leq \mathbf{B}\} \neq \emptyset$ **do**

Let P^* be the set of solutions of the following program:

$$\begin{cases} \min_{p' \in P'} s_{p'} & \\ \text{s.t.} & \\ x_{p,i} \geq 0, & \forall p \in \mathcal{P}, \forall i \in \mathcal{N} \\ x_{p,i} = 0, & \forall p \in \mathcal{P}, \forall i \in \mathcal{N} \text{ s.t. } c \notin A_i \\ \sum_{i \in \mathcal{N}} x_{p,i} = c(p), & \forall p \in \pi \cup \{p'\} \\ \sum_{i \in \mathcal{N}} x_{p,i} = 0, & \forall p \notin \pi \cup \{p'\} \\ s_{p'} \geq \sum_{p \in \mathcal{P}} x_{p,i}, & \forall i \in \mathcal{N} \end{cases}$$

Choose p in P^* and add it to π.

return π.

Condorcet consistency

Definition:

A budget allocation π is said to be Condorcet consistent if for every other feasible budget allocation π', π dominates π' for at least $\frac{n}{2}$ agents.

[20] Shapiro and Talmon "A Participatory Democratic Budgeting Algorithm" (2017)

Condorcet consistency

Definition:

A budget allocation π is said to be Condorcet consistent if for every other feasible budget allocation π', π dominates π' for at least $\frac{n}{2}$ agents.

Under the *minmax* extension of the preferences, a Concorcet consistent budget allocation can be computed in polynomial time if one exists (it is Smith-consistent).

[20] Shapiro and Talmon "A Participatory Democratic Budgeting Algorithm" (2017)

Smith-consistent budgeting algorithm

Input: An instance $I = \langle \mathcal{N}, \mathcal{P}, \mathbf{B} \rangle$.
Output: A budget allocation π.
Let G be the project majority graph.
Set $\succ$ to be an empty ordering.
while $G \neq \emptyset$ **do**
    $S \leftarrow$ Schwartz-set(G).
    Append S to the end of $\succ$.
    $G \leftarrow G \backslash S$.
Set π to be the empty budget allocation.
Consider $\succ$ to be $P_1, \succ \cdots \succ P_z$.
for $0 \leq i \leq z$ **do**
    $P \leftarrow$ a maximal feasible subset of P_i closest to π_{-1}.
    Add the set P to π.
return π.

Combinatorial Participatory Budgeting

Incentive compatibility

The knapsack voting rule

The knapsack voting rule [4] has been proved to satisfy incentive compatibility properties. That is:

- With *l_1 utility model*: it is *strategy-proof* and *welfare maximizing*.
- With *additive preferences*: the voters' best response is *partially strategy-proof*.

[4] Goel, Krishnaswamy, Sakshuwong, and Aitamurto "Knapsack voting: Voting mechanisms for participatory budgeting" (2019)

4. Related Frameworks

Budgeted social choice

The budgeted social choice problem [21] is very similar to that of combinatorial but the costs are composed of a *fixed component* and a *variable one*.

They consider utility functions expressed through *positional scoring rules*.

[21] Lu and Boutilier "Budgeted social choice: From consensus to personalized decision making" (2011)

Fair public decision making

In the public decision making model [22] agents have to select exactly one *alternative* for each *issue* they are facing. They study fairness property similar to the core and show that maximizing the Nash social welfare satisfies or approximates the fairness criteria.

This is a restriction of the combinatorial PB model where each issue would correspond to a resource.

[22] Conitzer, Freeman, and Shah "Fair public decision making" (2017)

Fair allocation of indivisible public goods

In this problem [23] a subset of public goods are to be selected under some constraints. The constraints came be of three types:

- *Matroid constraints*: given a matroid over the set of public goods, the chosen goods should form a basis of the matroid. This generalized the constraints imposed by [22] and the multi-winner voting.
- *Matching constraints*: the public goods are disposed on a graph and the selected ones should form a matching in the graph.
- *Packing constraints*: a set of knapsack constraints in considered. This generalizes the Combinatorial Participatory Budgeting to settings with multiple resources.

They present polynomial approximation algorithms to compute core allocations.

[23] Fain, Munagala, and Shah "Fair allocation of indivisible public goods" (2018)
[22] Conitzer, Freeman, and Shah "Fair public decision making" (2017)

5. Promising directions

Other direction for the current model

- *Requirement for groups of agents*: investigating criteria that apply to groups of agents can lead to a better selection of the rules. Moreover, studying pre-existing groups of agents with different entitlements can also be interesting for real-world applications.
- *More complex preferences*: in participatory budgeting, interactions between projects are frequently encountered. This can not be modelled with additive preferences, more complex preferences models are required.
- *Repetitive participatory budgeting*: some projects might take a certain number of years to be achieved, subsequent PB can then modify what has been decided.
- *Communication issues*: as with any voting procedure, the amount of information required to proceed can be critical.

Enriching the model

- *Multidimensional constraints*: most of the works focus on a single resource while the decision making problem can be more complex.
- *Distributional constraints*: there could be distributional constraints over the project instead of the usual knapsack one.
- *Hybrid models*: some hybrid models with discrete and divisible resources can be investigated as a generalization of the taxonomy presented.
- *End-to-end models*: studying the first stage of the participatory budgeting can lead to more insights on the mechanisms used to compute a budget allocation.