Effort-Based Fairness—Equity of Resources—for Participatory Budgeting

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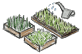
1. Introduction

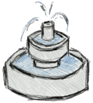
1000$

2000$

2500$



2500$



5000$

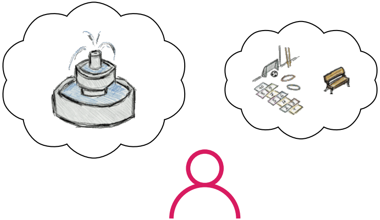
 : 7000$

7000$

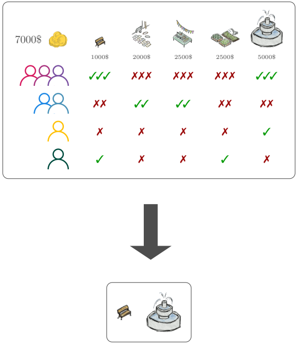
	1000$	2000$	2500$	2500$	5000$
	✓✓✓	✗✗✗	✗✗✗	✗✗✗	✓✓✓
	✗✗	✓✓	✓✓	✗✗	✗✗
	✗	✗	✗	✗	✓
	✓	✗	✗	✓	✗

Fairness Requirements

Incentive Compatibility



Algorithmic Perspective



Satisfaction-Based Fairness for Participatory Budgeting

Fairness is about distributing some *measure* fairly among the agents.

↳ What is a good measure in the case of participatory budgeting? *Satisfaction* is usually used.

Cardinal Utility Functions

- ✓ The satisfaction of an agent is obvious
- ✗ Hard to elicit
- ✗ Does not allow for interpersonal comparisons

Approval-Based Satisfaction

- ✓ Easy to elicit
- ✓ Has a clear meaning
- ✗ Unclear what proxy for satisfaction to use
  $|A \cap \pi|$ $c(A \cap \pi)$

We aim at *equity of resources* among the agents.

2. The Share

Definition

The share of an agent:
the ressources spent on
that specific agent

Cost of the project

$$share(\pi, A_i) = \sum_{p \in \pi \cap A_i} \frac{c(p)}{|\{A' \in \boldsymbol{A} \mid p \in A'\}|}$$

The budget allocation

The agent's ballot

Number of voters
approving of p

An Example

= 10

	1	2	3	4	5	
Cost	6	2	2	4	5	**Share**
	✓	✓		✓	✓	4
	✓		✓			2
	✓			✓		4

3. Providing Fair Share

The Perfect Situation

Every agent is provided their *fair share*, *i.e.*:

$$share(\pi, A_i) \geq \min \left\{ share(A_i, i), \frac{b}{n} \right\}$$

= 10	1	2	3	4	5	
Cost	6	2	2	4	5	**(Fair) Share**
	✓	✓		✓	✓	4; 3 + 1/3
	✓		✓			2; 3 + 1/3
	✓			✓		4; 3 + 1/3

A First Problem

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	4 / 3 + 1/3
	✓		✓			4 / 3 + 1/3
	✓			✓		2 / 3 + 1/3

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	4 / 3 + 1/3
	✓		✓			2 / 3 + 1/3
	✓			✓		4 / 3 + 1/3

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	4 / 3 + 1/3
	✓		✓			2 / 3 + 1/3
	✓			✓		2 / 3 + 1/3

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	7 / 3 + 1/3
	✓		✓			2 / 3 + 1/3
	✓			✓		0 / 3 + 1/3

= 10

Cost	6	2	2	4	5	(Fair) Share
	✓	✓		✓	✓	7 / 3 + 1/3
	✓		✓			0 / 3 + 1/3
	✓			✓		2 / 3 + 1/3

It is not possible to always provide fair share to everyone (and hard to know if we can).

4. Approximate Fair Share

Two Relaxations — Fair Share up to One Project

Every agent is provided their *fair share up to one project*, *i.e.*, for each agent there exists a project $p \in \mathcal{P}$ such that:

$$share(\pi \cup \{p\}, A_i) \geq \min \left\{ share(A_i, i), \frac{b}{n} \right\}$$

↳ This is however still unsatisfiable (and hard again)...

= 5	1	2	3
Cost	3	3	3
	✓	✓	
	✓		✓
		✓	✓

Two Relaxations — Local Fair Share

A budget allocation π provides *local fair share* if there is no project $p \in \mathcal{P} \setminus \pi$ such that for every agent i approving of p we have:

$$share(\pi \cup \{p\}, A_i) < \min \left\{ share(A_i, i), \frac{b}{n} \right\}$$

↳ An explanation? If such a p exists, all supporters of p receive less than their fair share and:

- Either p can be selected without exceeding the budget limit; let's select it then!
- Or, some voter $i^\star$ received more than their fair share; let's then exchange a project approved by $i^\star$ with p!

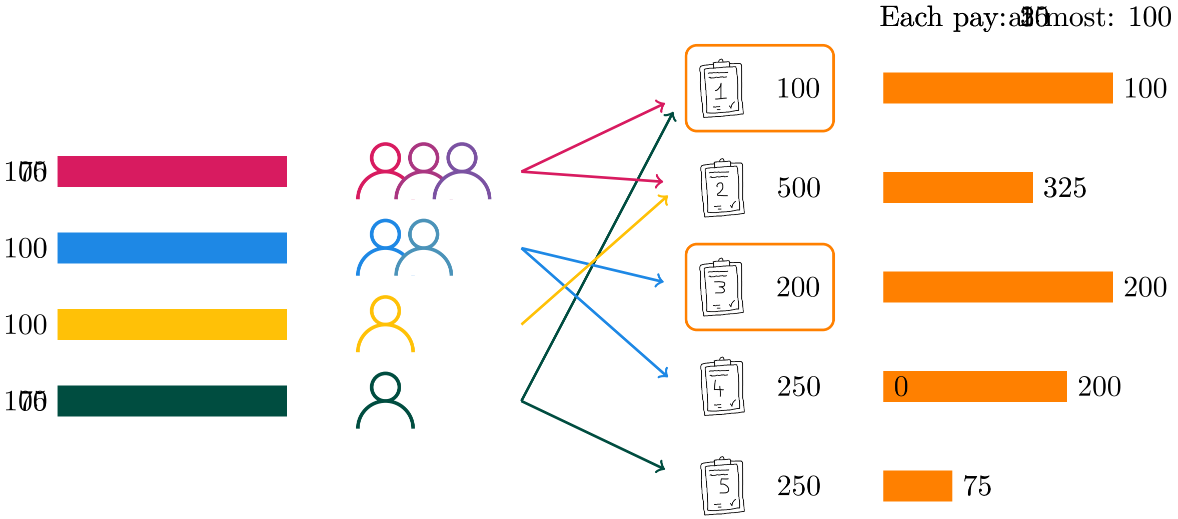
Note: This can be seen as a quota property: you add projects such that no one exceed their fair share as long as possible.

Finally Some Good News

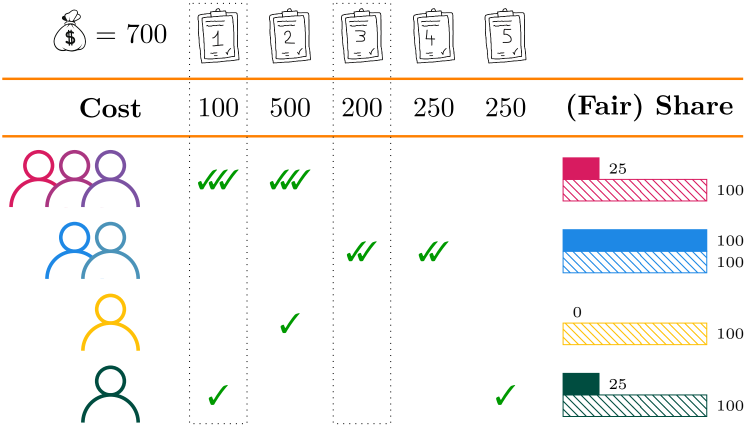
Local fair share is always satisfiable (and in polynomial time)!

↳ We can prove that *Rule X* (a.k.a. the method of equal share) satisfies local fair share.

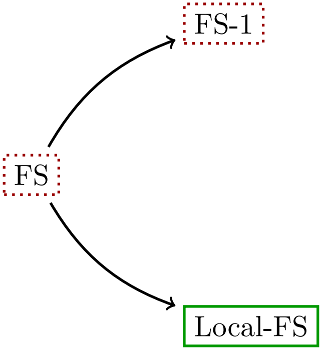
Rule X


Each pay: at most: 100
100
500
200
250
250
100
325
200
0
200
75
100
100
100
100

Rule X Satisfies Local Fair Share



The Picture so Far



5. Justified Share

Cohesive Groups

New idea: We want to provide what is deserved by the agents! But **what** do they deserve and **who**?

↳ Cohesive groups deserve to be represented to the amount of budget they control!

Agents in $N \subseteq \mathcal{N}$ are *P-cohesive*, if

$$\underbrace{P \subseteq \bigcap_{i \in N} A_i}_{\text{They are similar}} \quad \text{and} \quad \underbrace{\frac{|N|}{n} \geq \frac{c(P)}{b}}_{\text{They control enough units of budget}}$$

Providing Agents What They Deserve

Strong EJS: for every P-cohesive group N, for every agent $i \in N$, $\mathit{share}(\pi, i) \geq \mathit{share}(P, i)$.

Unsatisfiable

EJS: for every P-cohesive group N, there is an agent $i \in N$ such that $\mathit{share}(\pi, i) \geq \mathit{share}(P, i)$.

Satisfiable In Exponential Time

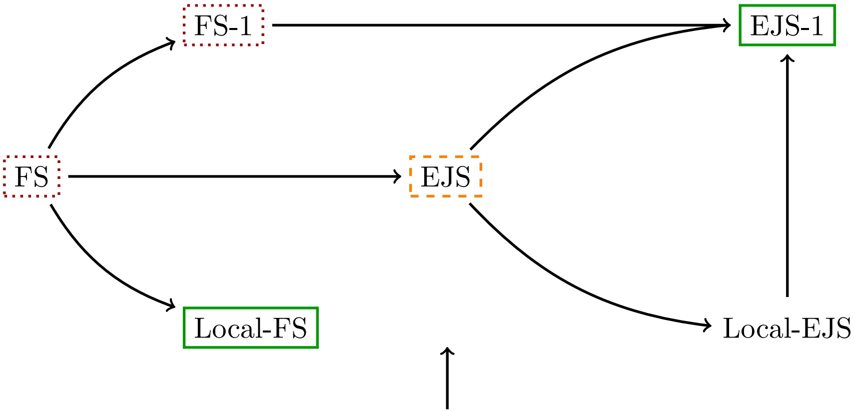
EJS-1: for every P-cohesive group N, there is an agent $i \in N$ and a project $p \in \mathcal{P}$ such that $\mathit{share}(\pi \cup \{p\}, i) \geq \mathit{share}(P, i)$.

Satisfiable In Polynomial Time

Local-EJS: for no P-cohesive group N would there exist a project $p \in P \setminus \pi$ such that for all agent $i \in N$, $\mathit{share}(\pi \cup \{p\}, i) < \mathit{share}(P, i)$.

Satisfiable Unknown for PB instances

Updated Picture



The arrow is proved to be missing here

6. Experimental Analysis of the Share

Experimental Analysis

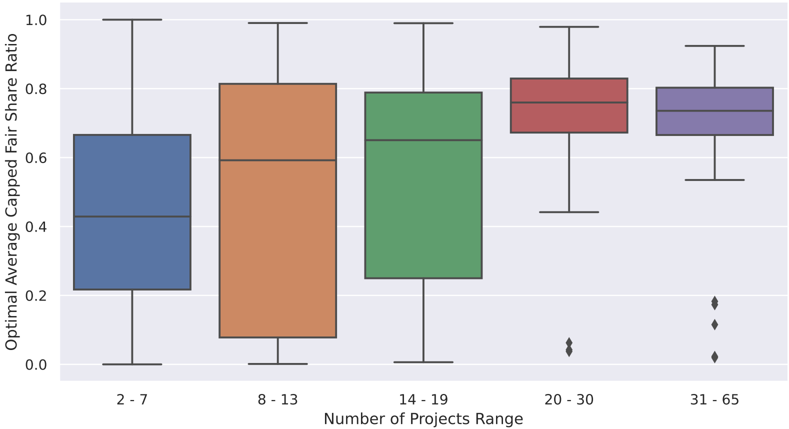
Instances: 350 instances from Pabulib with up to 65 projects.

Measure of Interest: The capped fair share ratio:

$$\min\left\{\frac{share(\pi, i)}{\min\{b/n, share(A_i, i)\}}, \ 1\right\}$$

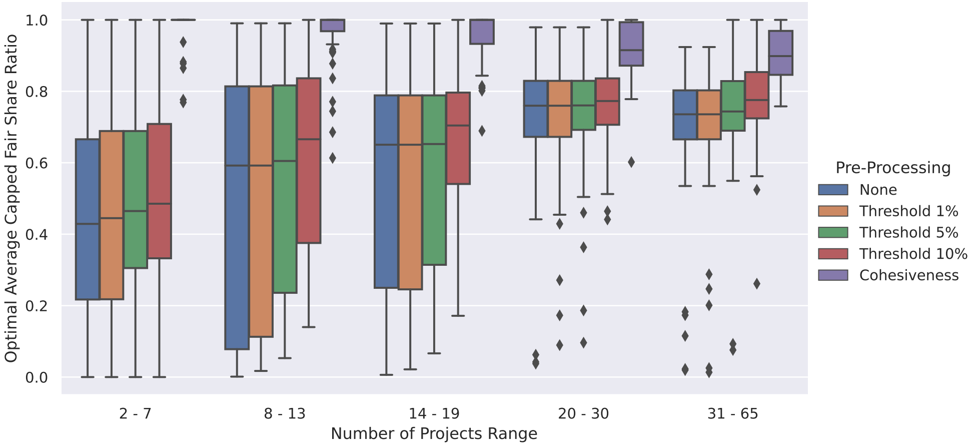
Fair share can be provided in only one instance out of the 350 considered (with 3 projects and 198 voters).

Optimal Average Fair Share Ratio



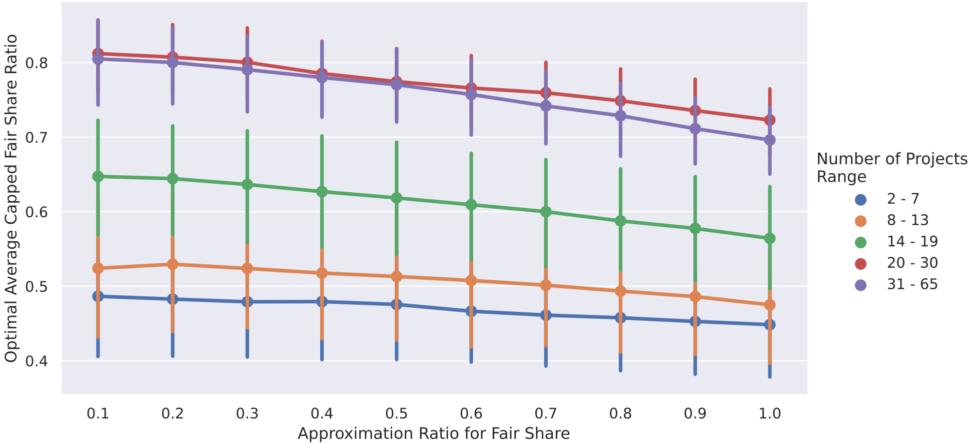
- We are far from achieving fair share.
- It gets easier as the number of projects increase.

Optimal Average Fair Share Ratio – Preprocessing



↳ Fair Share is hard to satisfy, structurally hard.

Optimal Average Fair Share Ratio – Apprximation



↳ Fair Share is hard to satisfy, structurally hard.

7. Conclusion

Wrap-Up

We have...

- ...Argued for defining fairness in terms of effort;
- ...Presented the share, one operationalisation of the idea of effort;
- ...Discussed how to satisfy fairness criteria related to the share.

Future work includes:

- Solving the Local-EJS matter (is it satisfiable in polynomial time?);
- Looking for non-sequential rules that could provide strong requirements (when they exist), *e.g.*, rules optimizing for fair share;
- Extending the experimental section: can we provide satisfaction-based and effort-based fairness at the same time?

Thanks!