Credulous Acceptability, Poison Game and Modal Logic

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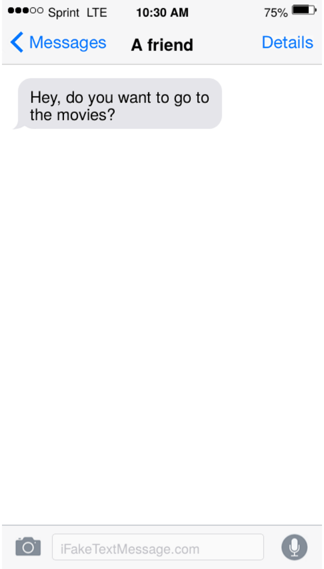
Plan

Argumentation framework



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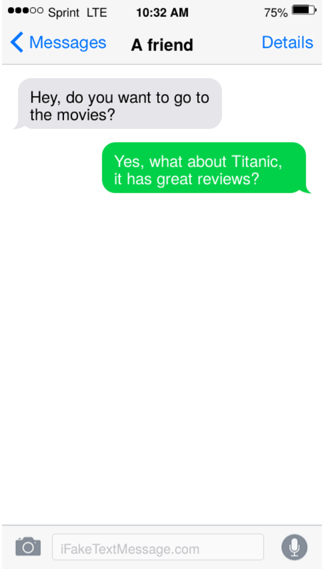
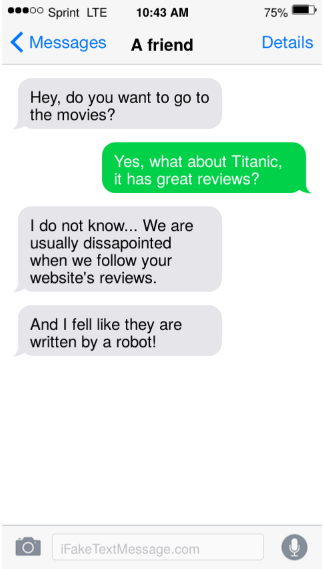
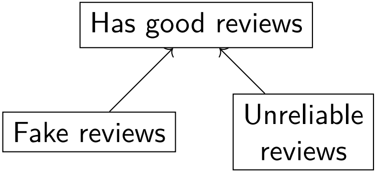


Has good reviews

Argumentation framework





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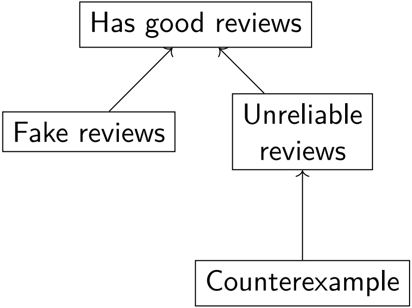

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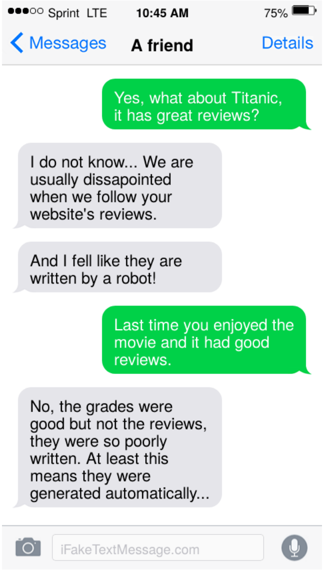

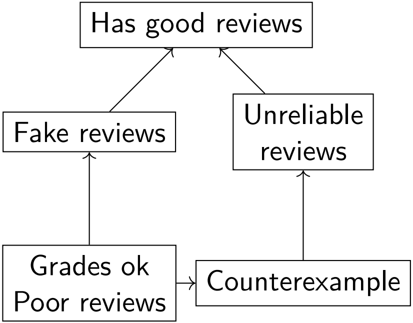

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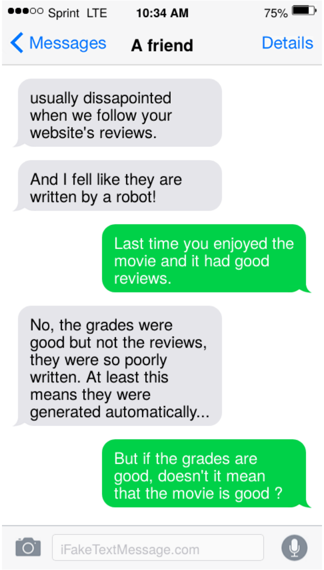
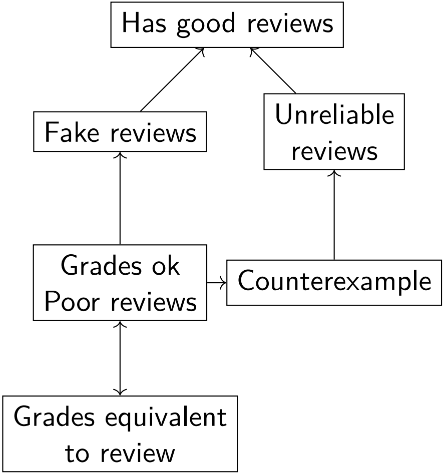
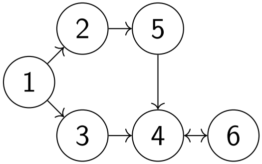




Argumentation framework

Definition (Argumentation framework)

An argumentation framework, is a graph $G = (A, \to)$ where A is a set of arguments and $\to \subseteq A^2$ is a set of attacks.

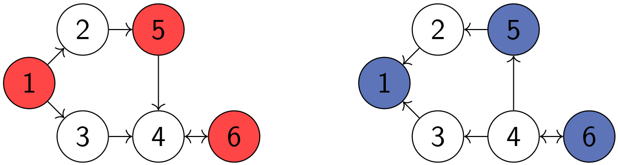


Admissible sets

Definition (Admissible set)

Let $G = (A, \rightarrow)$ be an argumentation framework, $X \subseteq A$ is an admissible set if:

- No two nodes in X attack one another
- For each $y \in A \backslash X$ such that $\exists x \in X, (y, x) \in \rightarrow$ then $\exists z \in X, (z, y) \in \rightarrow$.



An admissible set is also called a set of credulously acceptable arguments, it corresponds to a semi-kernel in the reverse graph.

Importance of admissible sets

- Main argumentation semantics[1]: a preferred extension is a maximal admissible set, this semantics generalizes the Reiter's extension semantics of default reasoning
- Graph-theoretic systematization of logic programming and default reasoning[2]: they correspond to partial stable models[3]
- Benchmark semantics for the evaluation of arguments[4]

[1] Dung, 1995.
[2] Dimopulos and Magirou, 1994.
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➥ Existence of admissible sets is a key reasoning task

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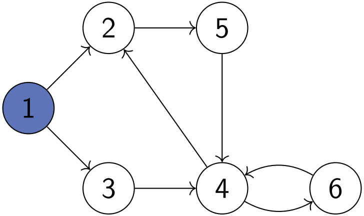
The Poison Game

A 2-player game[5] ($\mathbb{P}$ and $\mathbb{O}$) in which players successively move a token on a graph.



$\mathbb{P}$ chooses the first node

[5] P. Duchet and H. Meyniel (1993). "Kernels in directed graphs: a poison game". In: *Discrete mathematics* 115.1-3, pp. 273–276.

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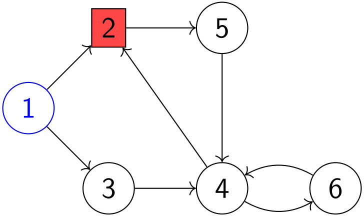

$\mathbb{O}$ moves and poisons 2

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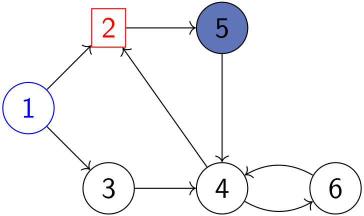

$\mathbb{P}$ goes to 5

[5] P. Duchet and H. Meyniel (1993). "Kernels in directed graphs: a poison game". In: *Discrete mathematics* 115.1-3, pp. 273–276.

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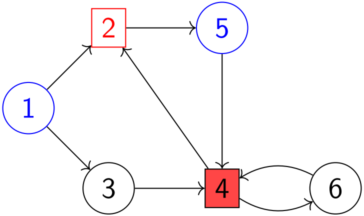

$\mathbb{O}$ poisons 4

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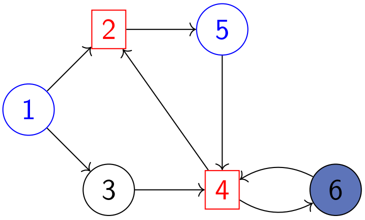

$\mathbb{P}$ indefinitely moves to 6 and wins the game

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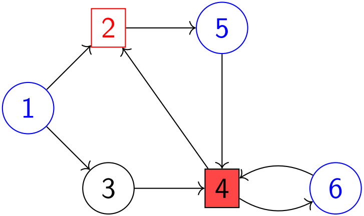

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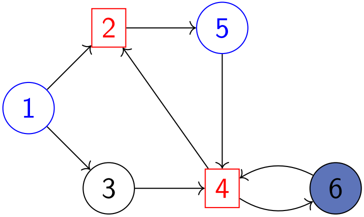

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Links between the Poison Game and Admissible Sets

Theorem (Duchet and Meyniel, 1993)

Let (W, R) be a finite directed graph. There exists a non-empty semi-kernel in (W, R) if and only if $\mathbb{P}$ has a winning strategy in the Poison Game for (W, R).

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➥ $\mathbb{P}$ has a winning strategy in the Poison Game for (W, R) if and only if there are credulously acceptable arguments in the argumentation framework (W, R^{-1})

Plan

Syntax

Definition (Poison modal language $\mathcal{L}^{\mathfrak{p}}$)

A formula of the poison modal language $\mathcal{L}^{\mathfrak{p}}$ is defined accordingly to the following grammar in Backus-Naur Form (BNF):

$$\mathcal{L}^{\mathfrak{p}} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \lozenge\varphi \mid \blacklozenge\varphi,$$

where $p \in \mathbf{P} \cup \{\mathfrak{p}\}$ with $\mathbf{P}$ a countable set of propositional atoms and $\mathfrak{p}$ a distinguished atom called *poison atom*.

We call $\mathcal{L}^{\mathfrak{p}}_n$ a multi-modal variant of $\mathcal{L}^{\mathfrak{p}}$ with n distinct pairs $(\lozenge_i, \blacklozenge_i)$ of modalities each equipped with a distinct poison atom $\mathfrak{p}_i$.

Models for PML are Kripke structures of the form $\mathcal{M} = (W, R, V)$ with W a set of world, $R \subseteq W \times W$ an accessibility relation and $V : \mathbf{P} \cup \{\mathfrak{p}\} \to 2^W$ a valuation function.

Poison Operator

Definition (Poison operator)

We define a poison operator • on models which modifies the valuation of a model by adding a world to the valuation of the poison atom $\mathfrak{p}$. That is for a model $\mathcal{M} = (W, R, V)$:

$$\mathcal{M}^{\bullet}_{w} = (W, R, V)^{\bullet}_{w} = (W, R, V')$$
$$\text{with, } \forall p \in \mathbf{P}, V'(p) = V(p)$$
$$\text{and } V'(\mathfrak{p}) = V(\mathfrak{p}) \cup \{w\}.$$

Definition (Poison relation)

The poisoning relation $\xrightarrow{\bullet}$ between two models is defined as:

$$(\mathcal{M}, w) \xrightarrow{\bullet} (\mathcal{M}', w') \Longleftrightarrow wR^{\mathcal{M}}w' \text{ and } \mathcal{M}' = \mathcal{M}^{\bullet}_{w'}.$$

Semantics

Definition (Satisfaction relation)

The satisfaction relation of PML is defined recursively for a given pointed model $(\mathcal{M}, w)$ as follows:

$$(\mathcal{M}, w) \models p \iff w \in V(p), \forall p \in \mathbf{P} \cup \{\mathfrak{p}\}$$

$$(\mathcal{M}, w) \models \neg\varphi \iff (\mathcal{M}, w) \not\models \varphi$$

$$(\mathcal{M}, w) \models \varphi \wedge \psi \iff (\mathcal{M}, w) \models \varphi \text{ and } (\mathcal{M}, w) \models \psi$$

$$(\mathcal{M}, w) \models \Diamond\varphi \iff \exists v \in W, R(w, v), (\mathcal{M}, v) \models \varphi$$

$$(\mathcal{M}, w) \models \blacklozenge\varphi \iff \exists v \in W, R(w, v), (\mathcal{M}^{\bullet}_{v}, v) \models \varphi$$

Definition (Poison modal equivalence)

We define the poison modal equivalence $\overset{\mathfrak{p}}{\leftrightsquigarrow}$ as follow:

$$(\mathcal{M}, w) \overset{\mathfrak{p}}{\leftrightsquigarrow} (\mathcal{M}', w') \iff (\mathcal{M}, w) \models \varphi \Leftrightarrow (\mathcal{M}', w') \models \varphi.$$

Validities

Let us define the dual poison modality $\blacksquare\varphi := \neg\blacklozenge\neg\varphi$.

Let $p \in \mathbf{P}$ be an atom and $\varphi \in \mathcal{L}^{\mathfrak{p}}$ and $\psi \in \mathcal{L}^{\mathfrak{p}}$ two PML formulas, then the following formulas are valid in PML:

$$\blacksquare\mathfrak{p} \tag{1}$$

$$\Box\bot \leftrightarrow \blacksquare\bot \tag{2}$$

$$\Box\bot \rightarrow \blacksquare\varphi \tag{3}$$

$$\blacksquare p \leftrightarrow \Box p \tag{4}$$

$$\Box\mathfrak{p} \rightarrow (\blacksquare\varphi \leftrightarrow \Box\varphi) \tag{5}$$

$$\blacksquare(\varphi \rightarrow \psi) \rightarrow (\blacksquare\varphi \rightarrow \blacksquare\psi) \tag{6}$$

$$\blacksquare(\varphi \wedge \psi) \leftrightarrow (\blacksquare\varphi \wedge \blacksquare\psi) \tag{7}$$

$$\blacksquare\neg\varphi \rightarrow (\Box\bot \vee \neg\blacksquare\varphi) \tag{8}$$

Cycle detection

Proposition

Let $\mathcal{M} = (W, R, V)$ *be a PML model such that* $V(\mathfrak{p}) = \emptyset$*, then for* $n \in \mathbb{N}_{>0}$ *there exists* $w \in W$ *such that* $(\mathcal{M}, w) \models \blacklozenge(\delta_n)$ *if and only if there exists a cycle of length* n *in the frame* (W, R)*, with:*

$$\begin{aligned} \delta_1 &= \Diamond \mathfrak{p} \\ \delta_2 &= \Diamond(\neg \mathfrak{p} \wedge \delta_1) \\ &\vdots \\ \delta_n &= \Diamond(\neg \mathfrak{p} \wedge \delta_{n-1}) \end{aligned}$$

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$$\begin{aligned} \delta_1 &= \lozenge\mathfrak{p} \\ \delta_2 &= \lozenge(\neg\mathfrak{p} \wedge \delta_1) \\ &\vdots \\ \delta_n &= \lozenge(\neg\mathfrak{p} \wedge \delta_{n-1}) \end{aligned}$$

➥ PML is not bisimulation-invariant, its formulas are not preserved by tree-unravelings and it does not enjoy the tree model property.

Winning strategies for the Poison Game

Winning positions for $\mathbb{O}$ are defined by the following infinitary $\mathcal{L}^{\mathfrak{p}}$-formula:

$$\blacklozenge\Box\mathfrak{p} \vee \blacklozenge\Box\blacklozenge\Box\mathfrak{p} \vee \ldots \tag{9}$$

Dually, winning positions for $\mathbb{P}$ are defined by the following infinitary $\mathcal{L}^{\mathfrak{p}}$-formula:

$$\blacksquare\Diamond\neg\mathfrak{p} \wedge \blacksquare\Diamond\blacksquare\Diamond\neg\mathfrak{p} \wedge \ldots \tag{10}$$

Remark (Credulous acceptability and PML)

By Duchet and Meyniel's theorem, formula (10)*, interpreted on the inversion of an argumentation framework, expresses the property "there exist credulously acceptable arguments in the framework".*

Plan

First-Order Logic translation

Definition (FOL translation)

Let $\mathfrak{p}, p, q, \ldots$ be propositional atoms, we call $\mathfrak{P}, P, Q, \ldots$ their corresponding first-order predicate.
Let N be a finite set of variables, and x a designated variable, the translation $ST_x^N : \mathcal{L}^{\mathfrak{p}} \to \mathcal{L}$ is defined inductively as follows:

$$ST_x^N(p) = P(x), \forall p \in \mathbf{P}$$
$$ST_x^N(\neg\varphi) = \neg ST_x^N(\varphi)$$
$$ST_x^N(\varphi \wedge \psi) = ST_x^N(\varphi) \wedge ST_x^N(\psi)$$
$$ST_x^N(\Diamond\varphi) = \exists y \left(xRy \wedge ST_y^N(\varphi)\right)$$
$$ST_x^N(\blacklozenge\varphi) = \exists y \left(xRy \wedge ST_y^{N\cup\{y\}}(\varphi)\right)$$
$$ST_x^N(\mathfrak{p}) = \mathfrak{P}(x) \vee \bigvee_{y\in N} (y = x).$$

Correctness of the translation

Theorem

Let $(\mathcal{M}, w)$ be a pointed model and $\varphi \in \mathcal{L}^{\mathfrak{p}}$ a formula, we have:

$$(\mathcal{M}, w) \models \varphi \iff \mathcal{M} \models ST_x^{\emptyset}(\varphi)[x := w].$$

Poison Bisimulation

Definition (p-bisimulation)

Two pointed models $(\mathcal{M}_1, w_1)$ and $(\mathcal{M}_2, w_2)$ are said to be p-bisimilar, written $(\mathcal{M}_1, w_1) \overset{\mathfrak{p}}{\rightleftharpoons} (\mathcal{M}_2, w_2)$, if there exists $Z \subseteq W^{\mathcal{M}_1} \times W^{\mathcal{M}_2}$ such that $w_1 Z w_2$ and whenever wZv we have:

Atom For any atom $p \in \mathbf{P} \cup \{\mathfrak{p}\}$, $w \in V^{\mathcal{M}_1}(p)$ iff $v \in V^{\mathcal{M}_2}(p)$.

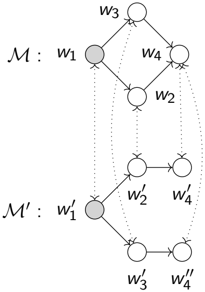
Zig$_\Diamond$ If there exists $w' \in \mathcal{M}_1$ such that $wR^{\mathcal{M}_1}w'$ then there exists $v' \in \mathcal{M}_2$ such that $vR^{\mathcal{M}_2}v'$ and $(\mathcal{M}_1, w')Z(\mathcal{M}_2, v')$.

Zag$_\Diamond$ If there exists $v' \in \mathcal{M}_2$ such that $vR^{\mathcal{M}_2}v'$ then there exists $w' \in \mathcal{M}_1$ such that $wR^{\mathcal{M}_1}w'$ and $(\mathcal{M}_1, w')Z(\mathcal{M}_2, v')$.

Zig$_\blacklozenge$ If there exists $(\mathcal{M}'_1, w'_1)$ such that $(\mathcal{M}_1, w_1) \overset{\bullet}{\to} (\mathcal{M}'_1, w'_1)$, then there exists $(\mathcal{M}'_2, w'_2)$ such that $(\mathcal{M}_2, w_2) \overset{\bullet}{\to} (\mathcal{M}'_2, w'_2)$ and $(\mathcal{M}'_1, w'_1)Z(\mathcal{M}'_2, w'_2)$.

Zag$_\blacklozenge$ If there exists $(\mathcal{M}'_2, w'_2)$ such that $(\mathcal{M}_2, w_2) \overset{\bullet}{\to} (\mathcal{M}'_2, w'_2)$, then there exists $(\mathcal{M}'_1, w'_1)$ such that $(\mathcal{M}_1, w_1) \overset{\bullet}{\to} (\mathcal{M}'_1, w'_1)$ and $(\mathcal{M}'_1, w'_1)Z(\mathcal{M}'_2, w'_2)$.

Example of p-bisimilar models



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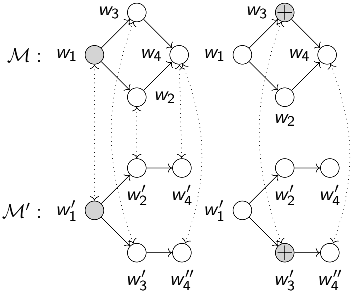

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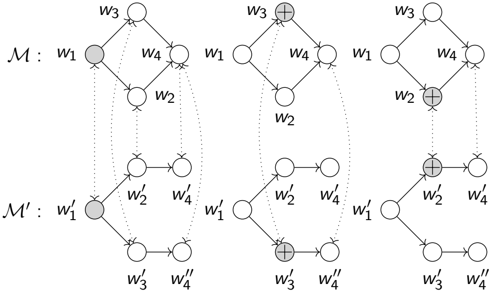

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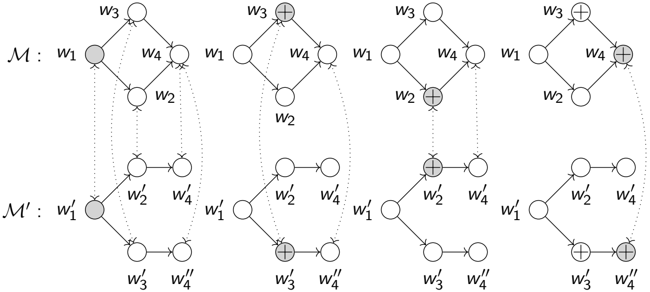
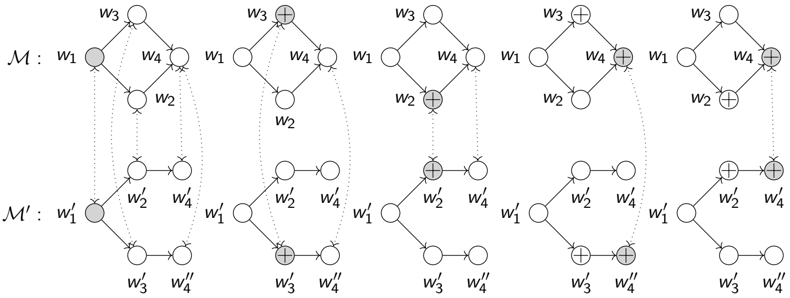


Example of p-bisimilar models



Characterization of PML

Theorem

For two pointed models $(\mathcal{M}_1, w_1)$ *and* $(\mathcal{M}_2, w_2)$*, if* $(\mathcal{M}_1, w_1) \overset{\mathfrak{p}}{\rightleftharpoons} (\mathcal{M}_2, w_2)$ *then* $(\mathcal{M}_1, w_1) \overset{\mathfrak{p}}{\leftrightsquigarrow} (\mathcal{M}_2, w_2)$.

Theorem

For any two ω*-saturated models* $(\mathcal{M}_1, w_1)$ *and* $(\mathcal{M}_2, w_2)$*, if* $(\mathcal{M}_1, w_1) \overset{\mathfrak{p}}{\leftrightsquigarrow} (\mathcal{M}_2, w_2)$ *then* $(\mathcal{M}_1, w_1) \overset{\mathfrak{p}}{\rightleftharpoons} (\mathcal{M}_2, w_2)$.

Theorem

An $\mathcal{L}$ *formula is equivalent to the translation of an* $\mathcal{L}^{\mathfrak{p}}$ *formula if and only if it is invariant for p-bisimulation.*

Interest of poison-bisimulation for argumentation theory

Remark (Credulous admissibility and p-bisimulation)

Formula (10) *expresses the existence of credulous admissible arguments, and is invariant for p-bisimulation.*

It directly follows that, given two p-bisimilar pointed models $(\mathcal{M}_1, w_1)$ *and* $(\mathcal{M}_2, w_2)$, *the frame of* $\mathcal{M}_1$ *contains credulously admissible arguments if and only if the frame of* $\mathcal{M}_2$ *does.*

Plan

Undecidability of PML_3

The satisfaction problem for PML_3 (PML with 3 modalities) can be defined as follows:

Data: A PML_3 formula $\varphi \in \mathcal{L}_3^{\mathfrak{p}}$.

Problem: Is there $(\mathcal{M}, w)$, with $V(\mathfrak{p}) = \emptyset$, s.t. $(\mathcal{M}, w) \models \varphi$?

Theorem

The satisfaction problem for PML_3 *is undecidable.*

The $\mathbb{N} \times \mathbb{N}$ tilling problem

Given a finite set of colors C, a tile is a 4-tuple of colors (its 4 sides). The $\mathbb{N} \times \mathbb{N}$ tilling problem is then defined as follows:

Data: A finite set T of tiles.

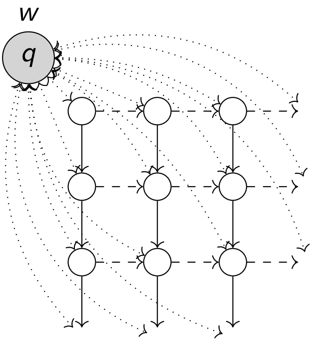
Problem: Can the infinite grid $\mathbb{N} \times \mathbb{N}$ be tiled using only tiles in T and such that two adjacent tiles share the same color on their common edge ?

This problem is known to be undecidable[6].

[6] D. Harel (1983). "Recurring dominoes: Making the highly undecidable highly understandable (preliminary report)". In: *International Conference on Fundamentals of Computation Theory*. Springer, pp. 177–194.

Proof of the undecidability of PML_3 - Example of a model

Let T be a finite set of tiles, φ_T[7] is satisfiable iff T tiles $\mathbb{N} \times \mathbb{N}$:



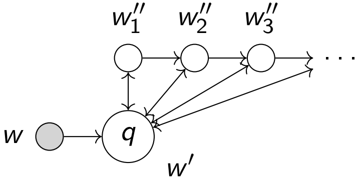
[7] B. Ten Cate and M. Franceschet (2005). "On the complexity of hybrid logics with binders". In: *International Workshop on Computer Science Logic*. Springer, pp. 339–354.

Failure of the finite model property of PML

Proposition

PML *does not have the finite model property.*

Let us consider φ_∞ such that all its models are infinite chains:



Plan

Memory Logic

The simplest memory logic[8], $\mathcal{M}(\textcircled{r}, \textcircled{k})$, extends modal semantics by considering frames (W, R, M) where $M \subseteq W$ is a set of states that have been 'memorized'. Its language is defined by:

$$\mathcal{L}_{\mathcal{M}} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Diamond\varphi \mid \textcircled{r} \mid \textcircled{k}$$

The semantics for the two new operators $\textcircled{r}$ and $\textcircled{k}$ is:

$$((W, R, M, V), w) \models \textcircled{r}\varphi \iff ((W, R, M \cup \{w\}, V), w) \models \varphi$$
$$((W, R, M, V), w) \models \textcircled{k} \iff w \in M,$$

where V is a valuation function.

➥ PML is a proper fragment of $\mathcal{M}(\textcircled{r}, \textcircled{k})$.

[8] C. Areces, D. Figueira, and S. Mera (2008). "Expressive power and decidability for memory logics". In: *Proceedings of WoLLIC 2008*. Vol. 5110. LNCS, pp. 56–68.

PML and Memory Logic

Proposition

$\mathcal{M}(ⓡ, ⓚ)$ *is strictly more expressive than* PML.

Proof.

An embedding of PML into $\mathcal{M}(ⓡ, ⓚ)$ can be defined as follow:

$$MT(\mathfrak{p}) = ⓚ$$
$$MT(\blacklozenge\varphi) = \Diamond ⓡ MT(\varphi)$$

PML and Memory Logic

Proposition

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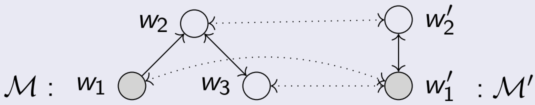
Proof.

An embedding of PML into $\mathcal{M}(ⓡ, ⓚ)$ can be defined as follow:

$$MT(\mathfrak{p}) = ⓚ$$
$$MT(\blacklozenge\varphi) = \Diamond ⓡ MT(\varphi)$$

Let $\mathcal{M}$ and $\mathcal{M}'$ defined below, we have $\mathcal{M}' \models ⓡ\Diamond\Diamond ⓚ$ while $\mathcal{M}$ falsifies it, but $\mathcal{M}$ and $\mathcal{M}'$ are p-bisimilar.



□

Hybrid Logic

The Hybrid Logic[9] $\mathcal{H}(\downarrow)$ is defined by the following grammar:

$$\mathcal{L}^{\mathcal{H}(\downarrow)} : \varphi := p \mid i \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Diamond\varphi \mid \downarrow x.\varphi,$$

with $p \in \mathbf{P} \cup \{\mathfrak{p}\}$ a propositional atom, and $i \in \mathbf{N}$ a nominal. Given an assignment $g : \mathbf{N} \to W$, g^x_m is called a x-variant of g if $\forall i \in \mathbf{N}, g(i) = g^x_m(i)$ and $g^x_m(x) = m$. The semantics is then defined as follows:

$$(M, g, m) \models_{\mathbf{H}} i \Leftrightarrow m = g(i)$$
$$(M, g, m) \models_{\mathbf{H}} \downarrow x.\varphi \Leftrightarrow (M, g^x_m, m) \models_{\mathbf{H}} \varphi$$

➥ PML can be embedded into $\mathcal{H}(\downarrow)$.

[9] P. Blackburn, J. van Benthem, and F. Wolter (2006). *Handbook of modal logic*. Vol. 3. Elsevier.

Hybrid Translation for PML

Let $HT^S : \mathcal{L}^{\mathfrak{p}} \to \mathcal{L}^{\mathcal{H}(\downarrow)}$, $S \subseteq \mathbf{N}$, be the translation defined as follows:

$$
\begin{aligned}
HT^S(p) &= p \\
HT^S(\mathfrak{p}) &= \mathfrak{p} \vee \bigvee_{i \in S} i \\
HT^S(\neg\varphi) &= \neg HT^S(\varphi) \\
HT^S(\varphi \wedge \psi) &= HT^S(\varphi) \wedge HT^S(\psi) \\
HT^S(\Diamond\varphi) &= \Diamond HT^S(\varphi) \\
HT^S(\blacklozenge\varphi) &= \Diamond\left(\downarrow x. HT^{S \cup \{x\}}(\varphi)\right)
\end{aligned}
$$

Proposition

Let $\mathcal{M} = (W, R, V)$ be a PML*-model, $M = (W, R, V')$ its hybrid extension, g an assignment and $\varphi \in \mathcal{L}^{\mathfrak{p}}$ a* PML*-formula, we have:*

$$(\mathcal{M}, w) \models \varphi \iff (M, g, w) \models_{\mathbf{H}} HT^{\emptyset}(\varphi).$$

Inclusion between PML and other logics

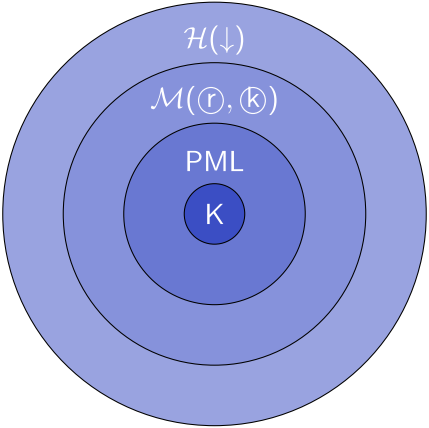


Figure: Links between PML and other logics

Plan

Summary of the presentation

We introduced and studied a modal logic PML that arises naturally from a game-theoretic approach to a central decision problem in argumentation theory: the existence of credulously acceptable arguments. We presented:

- A First-Order Logic translation
- An adequate bisimulation definition
- The undecidability of PML_3
- The links between PML and other logics

Future work

Concerning PML, some questions are left open:

- Can it be axiomatized?
- Is PML with one modality decidable?

In a broader view, this logic (like Sabotage Logic) calls for fixpoint extensions which pose interesting challenges[10].

From the argumentation perspective, it could be interesting to have a look to skeptical semantics, i.e. arguments that belong to all admissible sets of a framework.

For further discussion:

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Thank you !

[10] G. Aucher, J. van Benthem, and D. Grossi (2017). "Modal logics of sabotage revisited". In: *Journal of Logic and Computation* 28.2, pp. 269–303.

Proof of the undecidability of PML_3 - Grid structure

- w is a q-world, its R-successors are not q and link back to it, and the set of its R-successors is closed under R_1 and R_2:
  $\alpha = q \wedge \Box(\neg q \wedge \Diamond q) \wedge \Box\blacksquare_1\Diamond(q \wedge \Diamond\mathfrak{p}) \wedge \Box\blacksquare_2\Diamond(q \wedge \Diamond\mathfrak{p})$
- For all R-successor of w, accessibility relations R_1 and R_2 are total functions:
  $$\beta = \bigwedge_{i=1,2} (\Box\Diamond_i\top \wedge \blacksquare\Box(q \to \Box(\Diamond_i\mathfrak{p} \to \Box_i\mathfrak{p})))$$
- Accessibility relations R_1 and R_2 commute:
  $$\gamma = \blacksquare\Box\,(q \to \Box(\Box_1\Box_2\neg\mathfrak{p} \vee \Box_2\Box_1\mathfrak{p}))$$

Proof of the undecidability of PML_3 - Correct tilling

- Only one tile is present at each node:

$$\delta_T^1 = \bigvee_{t \in T} \left(p_t \wedge \bigwedge_{t' \in T, t' \neq t} \neg p_{t'} \right)$$

- Horizontal and vertical tilling are correct:

$$\delta_T^2 = \bigwedge_{t \in T} \left[\left(p_t \to \Box_1 \bigvee_{t' \in T, \mathbf{b}(t') = \mathbf{t}(t)} p_t \right) \wedge \left(p_t \to \Box_2 \bigvee_{t' \in T, \mathbf{l}(t') = \mathbf{r}(t)} p_t \right) \right]$$

Failure of the finite model property of PML - φ_∞

- $\alpha = \neg q \wedge \Diamond\top \wedge \Box q \wedge \Box(\Diamond\top \wedge \Box\neg q)$: the current state falsifies q and all its successors (there exists at least one) are q and have in turn successors (at least one) which all falsify q.
- $\beta = \blacksquare\Box\Diamond\mathfrak{p}$: after any poisoning a state is reached whose successors can reach the poisoned state in one step. In other words, all successors of the current state have successors linked via symmetric edges.
- $\gamma = \blacksquare\Box\Diamond(\neg q \wedge \Diamond\mathfrak{p}) \wedge \Box\Box\blacksquare\Box\neg\mathfrak{p}$: after any poisoning a state is reached whose successors are not reflexive loops (right conjunct), and can reach a $\neg q$ state which can in turn reach the poison state. In other words, all successors of the current state lay on cycles of length 3.
- $\delta = \Box\Box\blacksquare\Box(q \to \Diamond\mathfrak{p})$: all successors of the current state's successors are such that after any poisoning, and further q-successor can reach back to the poisoned state.
- $\epsilon = \Box\blacklozenge\neg\Diamond(q \wedge \Diamond(\neg q \wedge \Diamond\mathfrak{p})))$: all successors of the current state are such that there is one successor that can be poisoned and such that none of its successors satisfies q and can reach the poisoned state in two steps via a $\neg q$ state.

Tableau method for PML

Input: A formula $\varphi \in \mathcal{L}^{\mathfrak{p}}$
Output: A tableau $\mathcal{T}$ for φ with each branch labeled closed or open.

(1) Initiate $\mathcal{T}$ with a single node (the root) labeled with (φ, x, ϵ)

(2) Repeat as long as there are rules that can be applied:

- (I) Choose a branch B that is not labeled "close" nor "open".
- (II) Choose a formula (ψ, x, s), or a pair (ψ, x, s) and xRx_1, in B that has not been selected before and for which a tableau rule R (Figure **??**) can be applied.
  - (A) If $R = \neg\wedge$ (resp $R = \Diamond$ or $R = \blacklozenge$), add 2 (resp. $n + 1$) successors to B labeled with the denominator of R.
  - (B) Else, add a single successor labeled with the denominator of R.
- (III) Analyze the branches:
  - (A) Label "close" a branch which contains:
    - (i) Either (p, x, s) and $(\neg p, x, s')$ where $p \in \mathbf{P} \cup \{\mathfrak{p}\}$.
    - (ii) Or $(\mathfrak{p}_{\blacklozenge}, x, s)$ and $(\neg\mathfrak{p}, x, s')$ where s' is a prefix of s.
  - (B) Label "open" a branch for which no rules can be applied.

Tableau method for PML

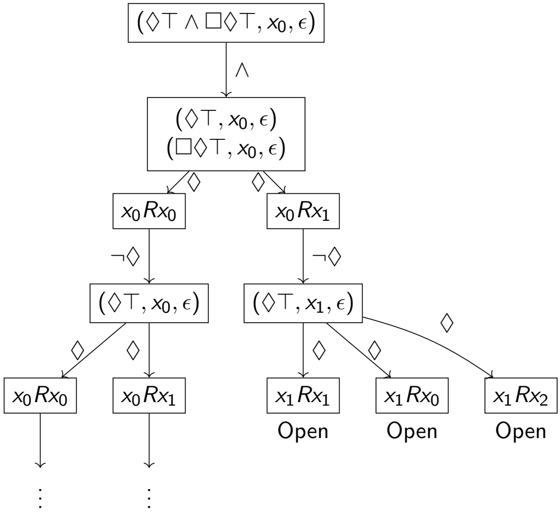
$$\frac{(\neg\neg\varphi, x, s)}{(\varphi, x, s)}\ \neg\neg \qquad \frac{(\varphi \wedge \psi, x, s)}{\begin{array}{c}(\varphi, x, s)\\ (\psi, w, s)\end{array}}\ \wedge \qquad \frac{(\neg(\varphi \wedge \psi), x, s)}{(\neg\varphi, x, s) \mid (\neg\psi, x, s)}\ \neg\wedge$$

$$\frac{(\Diamond\varphi, x, s)}{\begin{array}{c}xRx_1\\ (\varphi, x_1, s)\end{array} \Bigg| \cdots \Bigg| \begin{array}{c}xRx_n\\ (\varphi, x_n, s)\end{array} \Bigg| \begin{array}{c}xRx_0\\ (\varphi, x_0, s)\end{array}}\ \Diamond \qquad \frac{(\neg\Diamond\varphi, x, s) \quad xRx_1}{(\neg\varphi, x_1, s)}\ \neg\Diamond$$

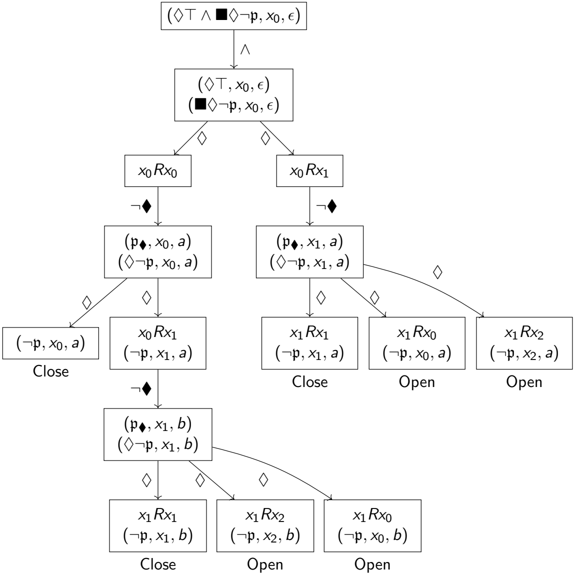
$$\frac{(\blacklozenge\varphi, x, s)}{\begin{array}{c}xRx_1\\ (\varphi, x_1, s \cdot a)\\ (\mathfrak{p}_{\blacklozenge}, x_1, s \cdot a)\end{array} \Bigg| \cdots \Bigg| \begin{array}{c}xRx_n\\ (\varphi, x_n, s \cdot a)\\ (\mathfrak{p}_{\blacklozenge}, x_n, s \cdot a)\end{array} \Bigg| \begin{array}{c}xRx_0\\ (\varphi, x_0, s \cdot a)\\ (\mathfrak{p}_{\blacklozenge}, x_0, s \cdot a)\end{array}}\ \blacklozenge \qquad \frac{(\neg\blacklozenge\varphi, x, s) \quad xRx_1}{\begin{array}{c}(\neg\varphi, x_1, s \cdot a)\\ (\mathfrak{p}_{\blacklozenge}, x_1, s \cdot a)\end{array}}\ \neg\blacklozenge$$

For rules $\Diamond$ and $\blacklozenge$, $\{x_1, \dots, x_n\}$ are all labels occurring in the current branch and x_0 is a fresh label not occurring in the current branch.
For rules $\blacklozenge$ and $\neg\blacklozenge$, $a \in \Gamma$ is a fresh letter of the alphabet that has never been used before.

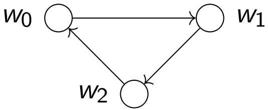
Example of infinite tableau



Example of finite tableau



Let us consider the following model $\mathcal{M}$:



The successive truth-set of $||\neg\mathfrak{p} \wedge \blacksquare\Diamond q||_{\mathcal{M}_{[q:=X]}}$ given the different values of X are given in the following table.

X	$\emptyset$	$\{w_0\}$	$\{w_1\}$	$\{w_2\}$	$\{w_0, w_1\}$	$\{w_0, w_2\}$	$\{w$
$\lvert\lvert\neg\mathfrak{p} \wedge \blacksquare\Diamond q\rvert\rvert_{\mathcal{M}_{[q:=X]}}$	$\emptyset$	$\{w_1\}$	$\{w_2\}$	$\{w_0\}$	$\{w_1, w_2\}$	$\{w_0, w_1\}$	$\{w$

We then have $\forall w \in W, (\mathcal{M}, w) \models \nu q. (\neg\mathfrak{p} \wedge \blacksquare\Diamond q)$, whereas no nodes in (W, R) is a winning position for $\mathbb{P}$.