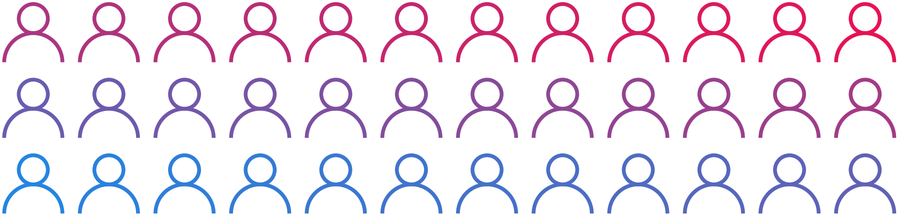
A Selective Literature Review of the Truth Tracking Approach in Computational Social Choice

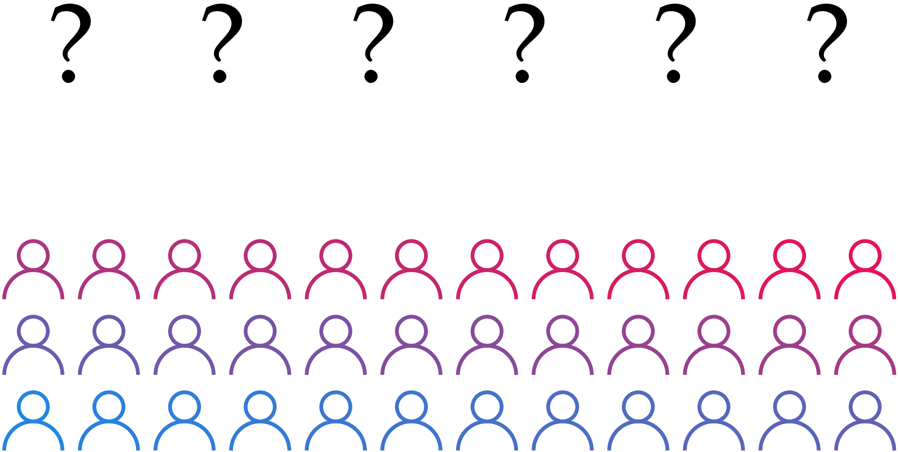
Simon Rey

November 26, 2020

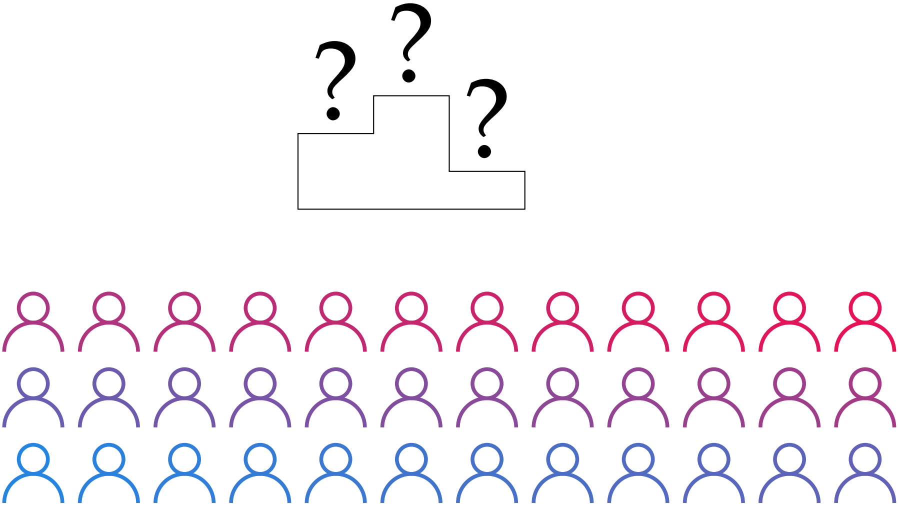
Voting Theory



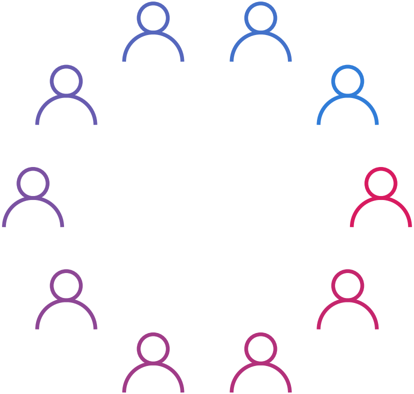
Voting Theory



Voting Theory



Two Views on Voting



Two Views on Voting

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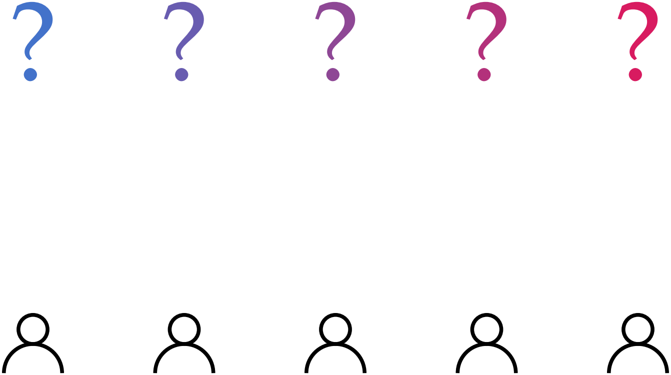
Axiomatic approach: Studying voting rules through the normative properties they satisfy.

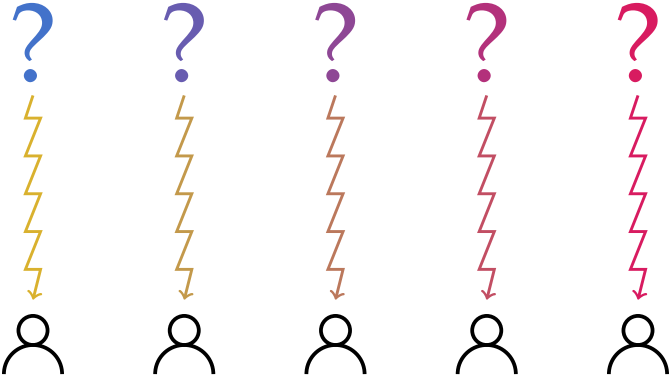
Two Views on Voting

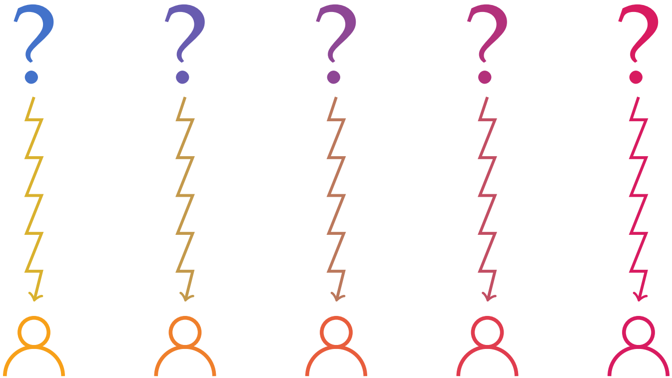
? ? ? ? ?

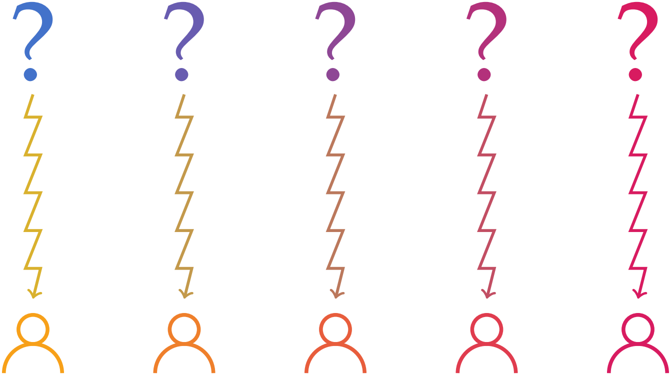
Two Views on Voting



Two Views on Voting







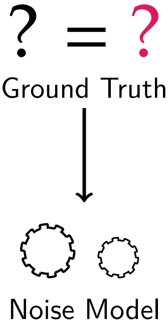
Epistemic approach: Studying voting rules through their ability to recover the ground truth.

Noise Models

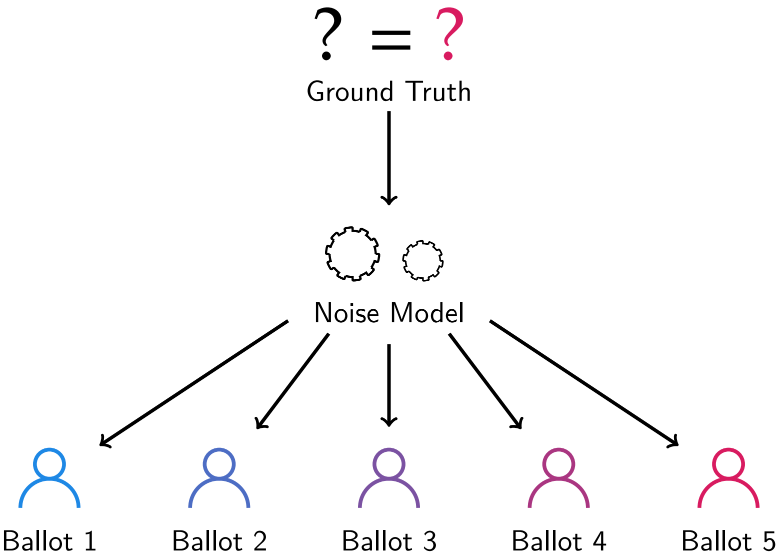
$$? = ?$$

Ground Truth

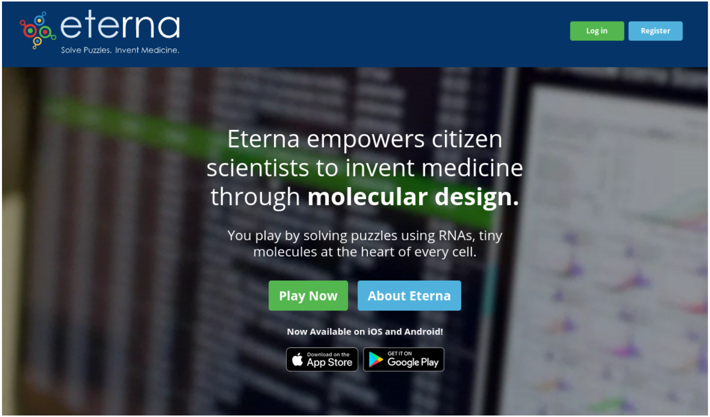
Noise Models



Noise Models



Ground Truth



amazon mechanical turk beta

Plan for the Day

- Condorcet Jury Theorem

Plan for the Day

- Condorcet Jury Theorem
- Maximum Likelihood Approach

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- Sample Complexity

Plan for the Day

- Condorcet Jury Theorem
- Maximum Likelihood Approach
- Sample Complexity
- Robusteness to noise

Plan for the Day

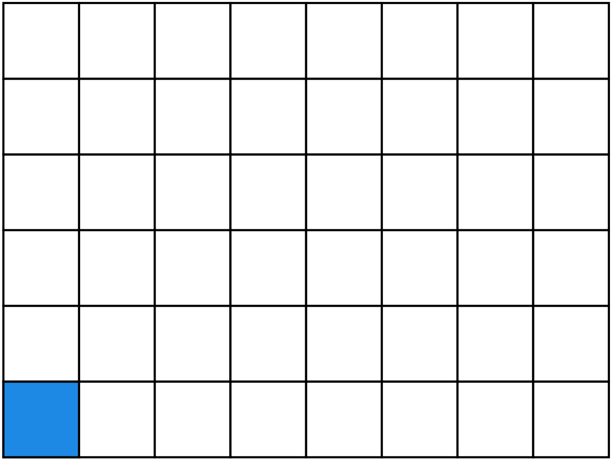
- Condorcet Jury Theorem
- Maximum Likelihood Approach
- Sample Complexity
- Robusteness to noise

From specific noise models
to classes of noise models

1. Simple Case: Two Candidates

Two Candidates Election with Uniform Prior

Accuracy: 60%

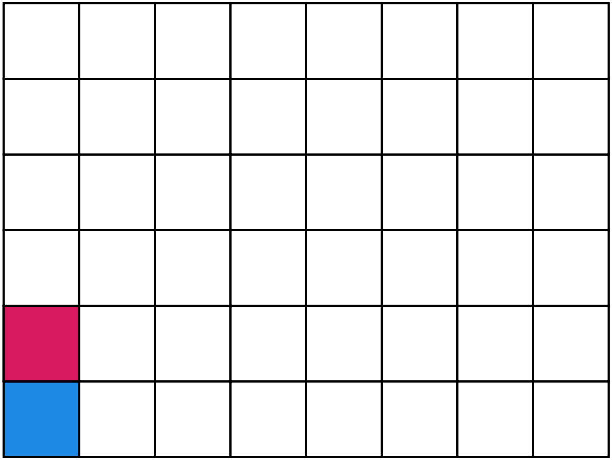


Votes for A: 0

Votes for B: 1

Two Candidates Election with Uniform Prior

Accuracy: 60%

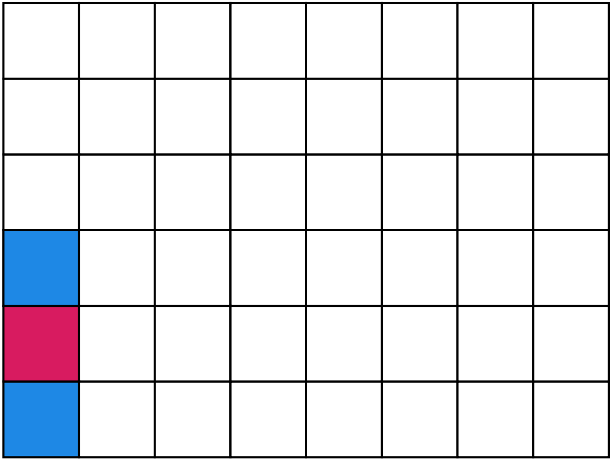


Votes for A: 1

Votes for B: 1

Two Candidates Election with Uniform Prior

Accuracy: 60%

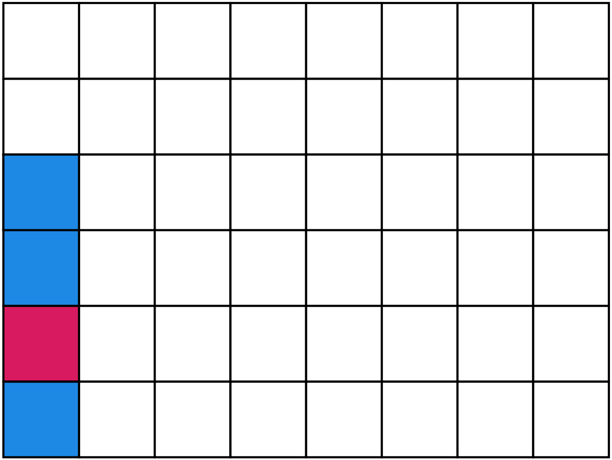


Votes for A: 1

Votes for B: 2

Two Candidates Election with Uniform Prior

Accuracy: 60%

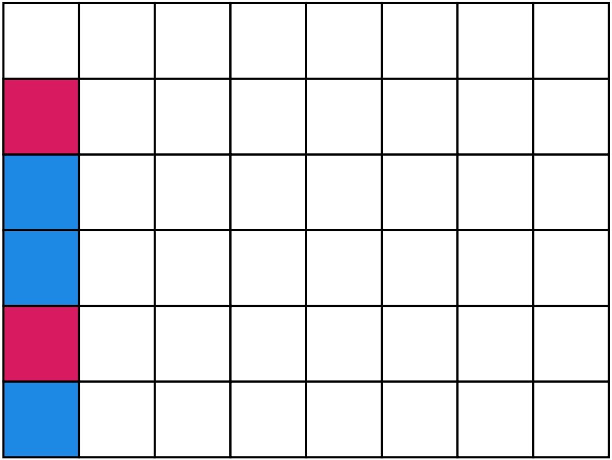


Votes for A: 1

Votes for B: 3

Two Candidates Election with Uniform Prior

Accuracy: 60%

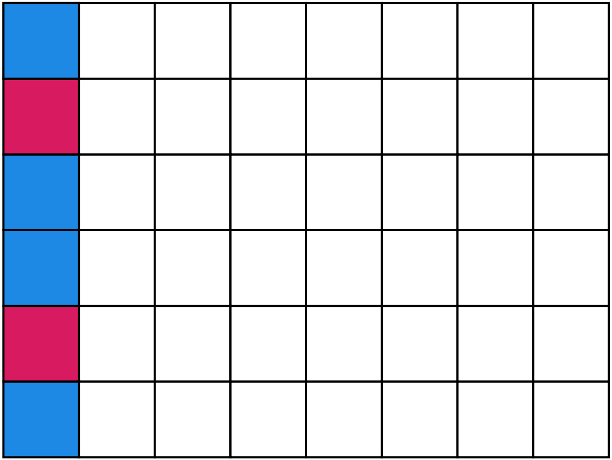


Votes for A: 2

Votes for B: 3

Two Candidates Election with Uniform Prior

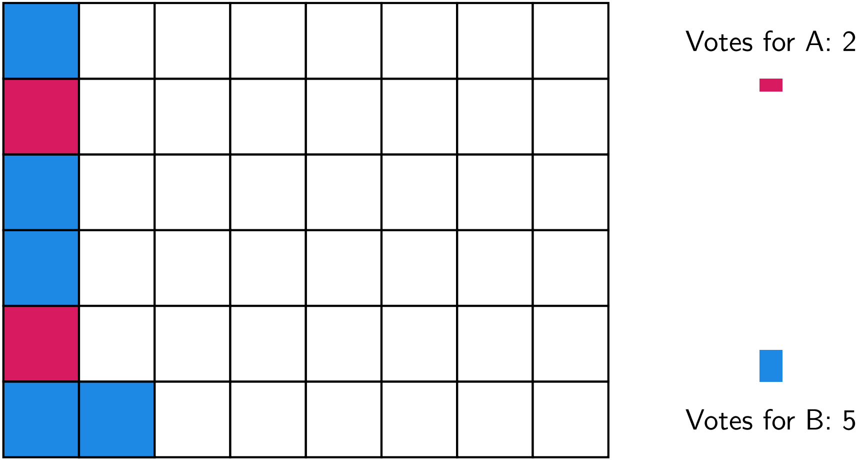
Accuracy: 60%



Votes for A: 2

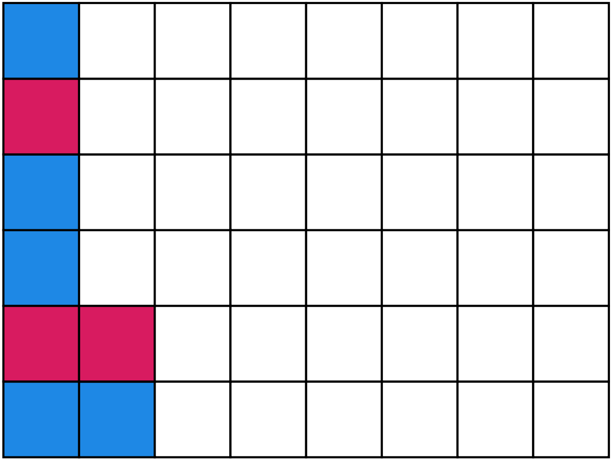
Votes for B: 4

Accuracy: 60%



Two Candidates Election with Uniform Prior

Accuracy: 60%

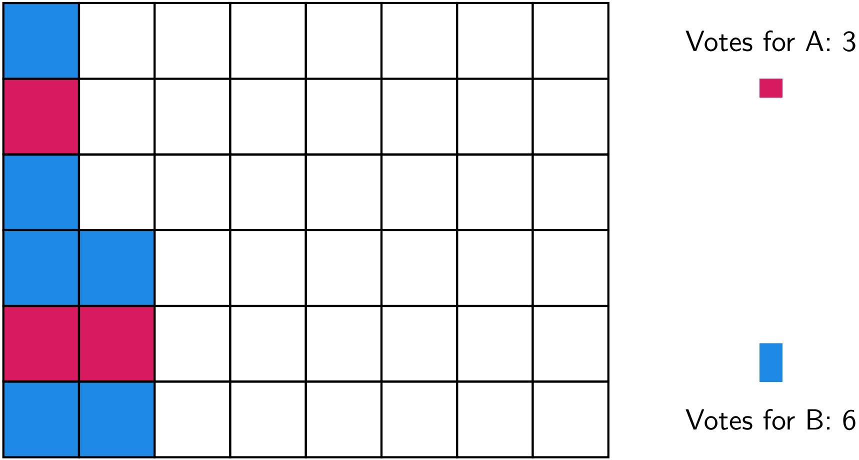


Votes for A: 3

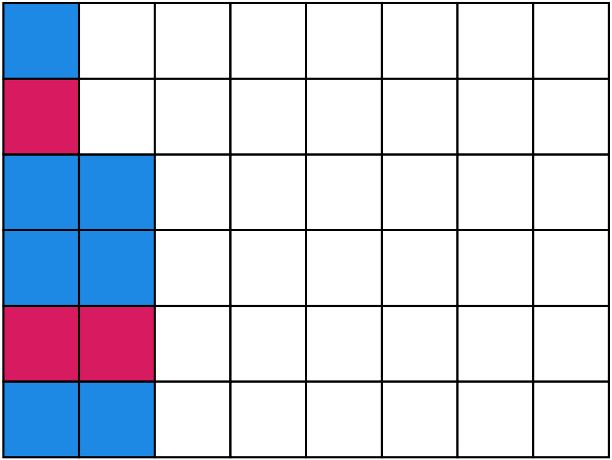
Votes for B: 5

Two Candidates Election with Uniform Prior

Accuracy: 60%



Accuracy: 60%

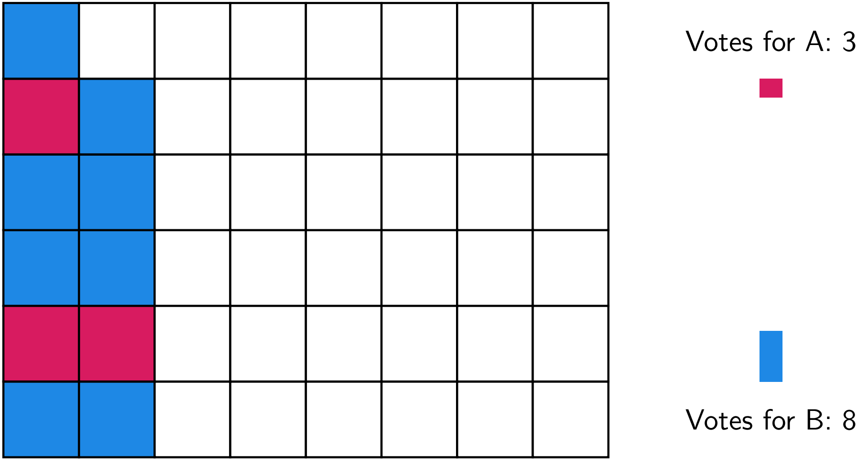


Votes for A: 3

Votes for B: 7

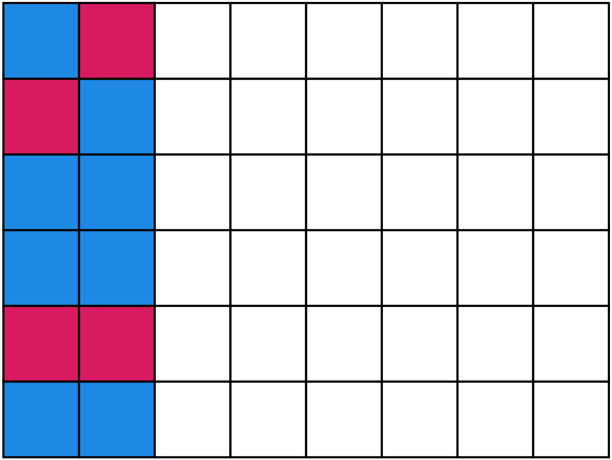
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

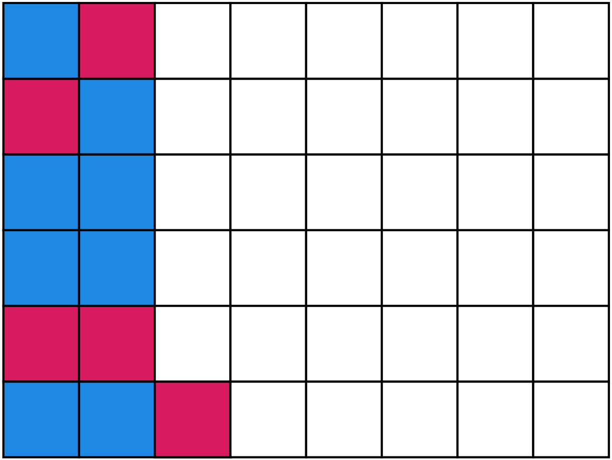
Accuracy: 60%



Votes for A: 4

Votes for B: 8

Accuracy: 60%

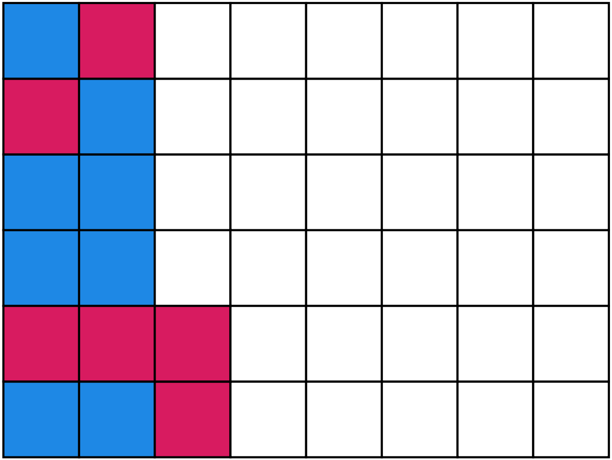


Votes for A: 5

Votes for B: 8

Two Candidates Election with Uniform Prior

Accuracy: 60%

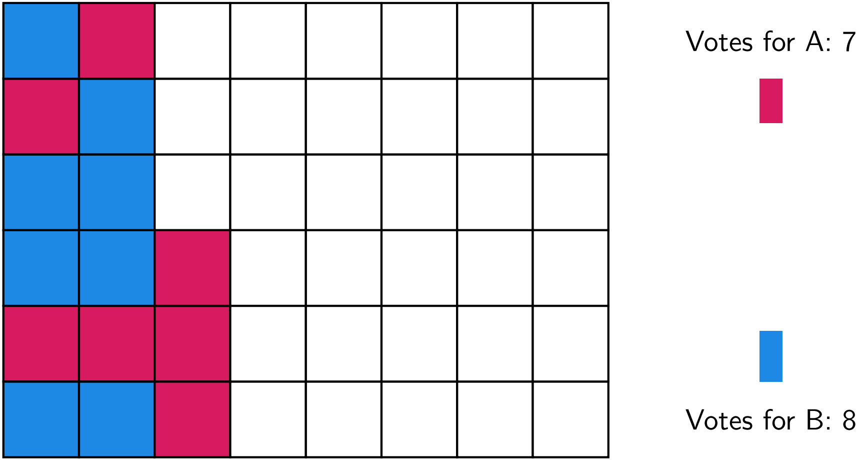


Votes for A: 6

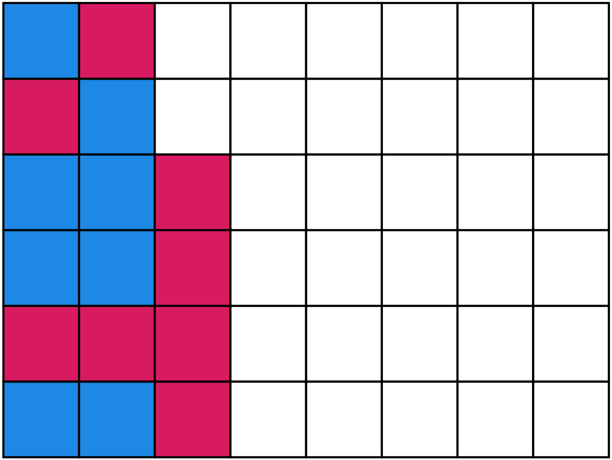
Votes for B: 8

Two Candidates Election with Uniform Prior

Accuracy: 60%



Accuracy: 60%

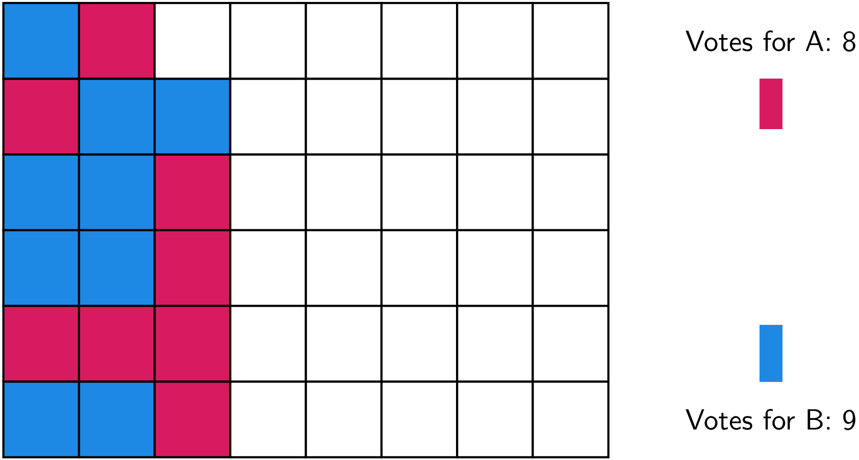


Votes for A: 8

Votes for B: 8

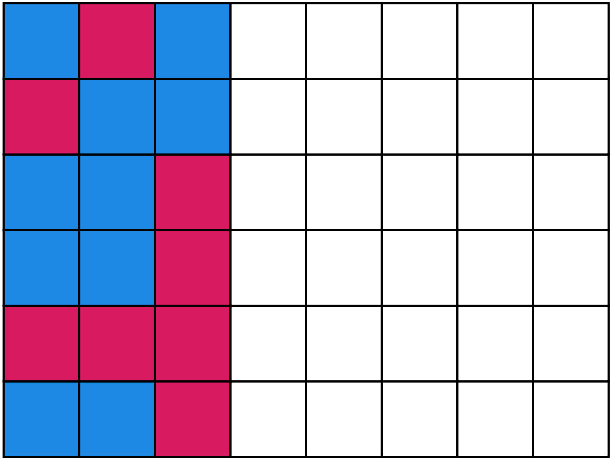
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

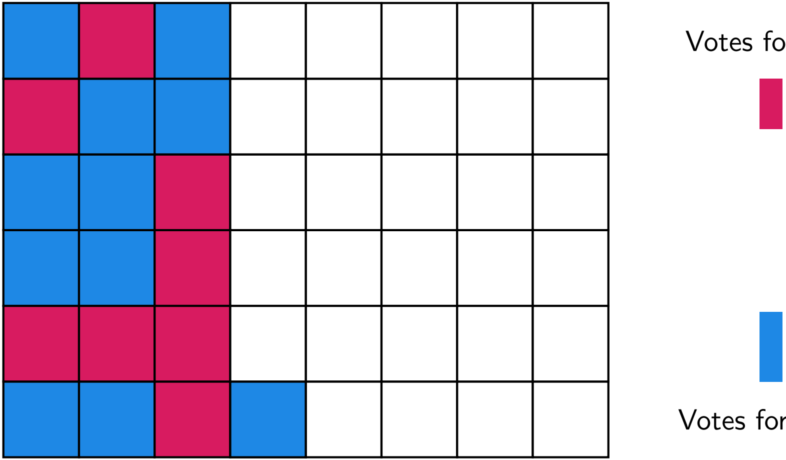
Accuracy: 60%



Votes for A: 8

Votes for B: 10

Accuracy: 60%

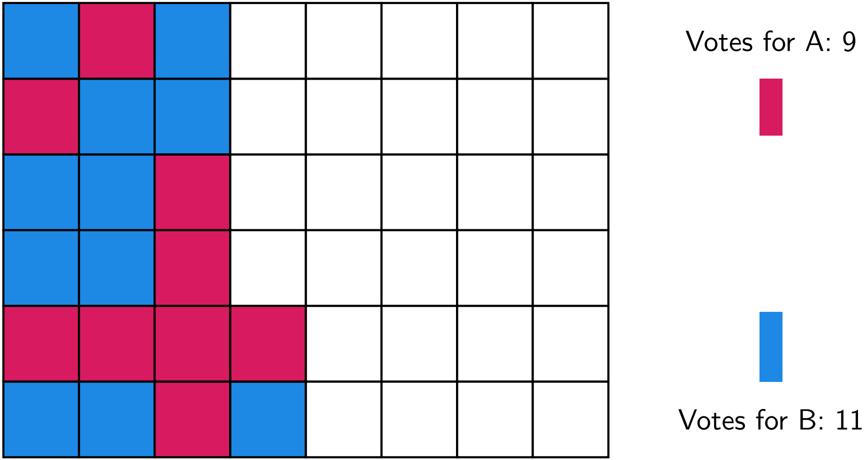


Votes for A: 8

Votes for B: 11

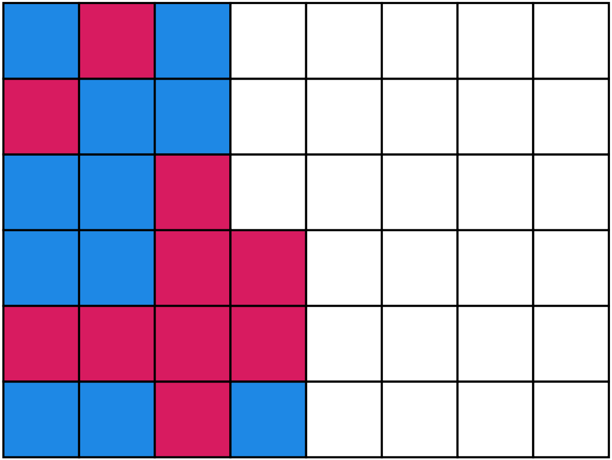
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

Accuracy: 60%

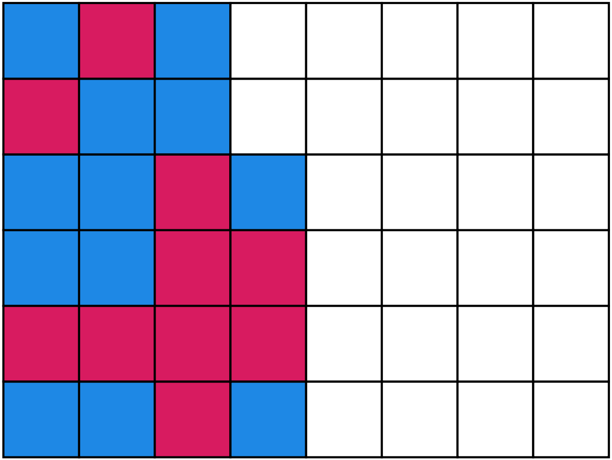


Votes for A: 10

Votes for B: 11

Two Candidates Election with Uniform Prior

Accuracy: 60%

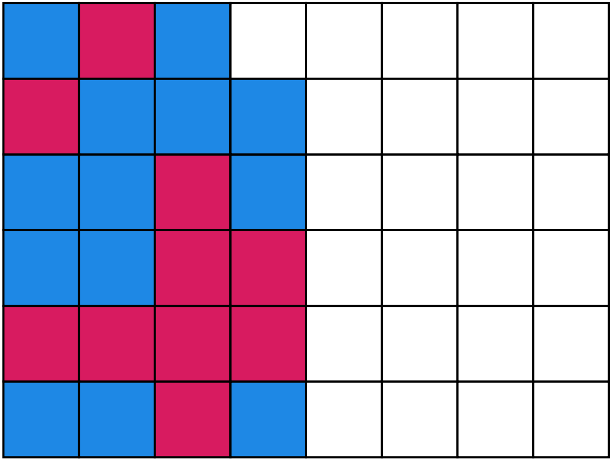


Votes for A: 10

Votes for B: 12

Two Candidates Election with Uniform Prior

Accuracy: 60%

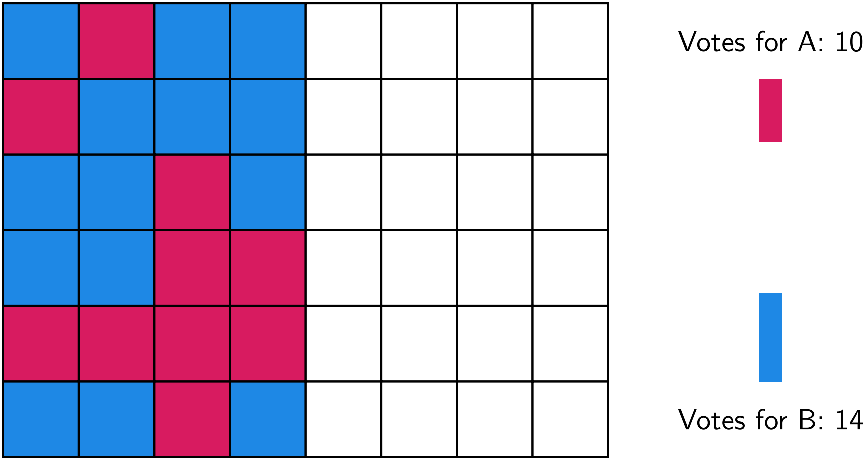


Votes for A: 10

Votes for B: 13

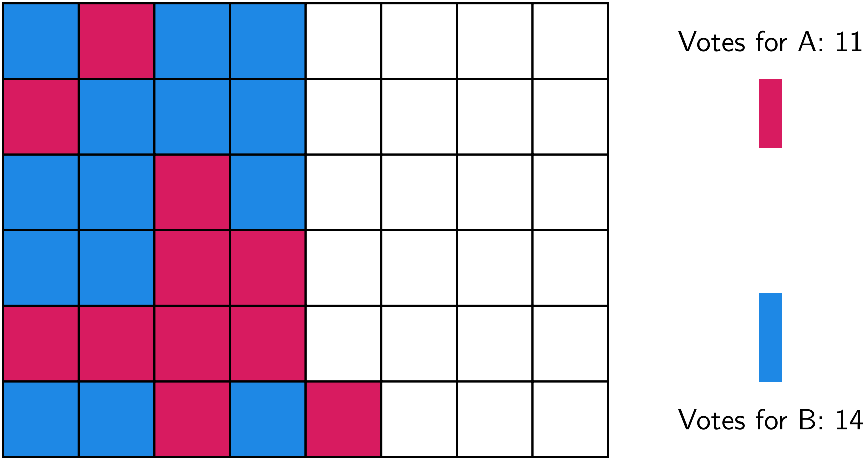
Two Candidates Election with Uniform Prior

Accuracy: 60%



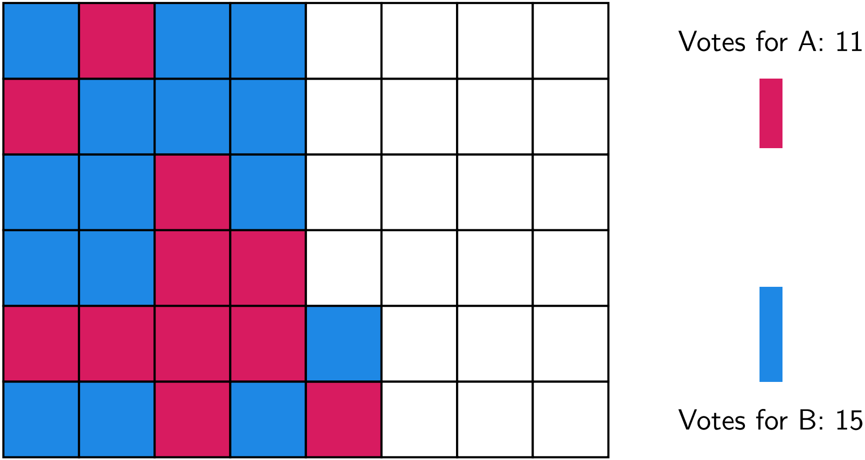
Two Candidates Election with Uniform Prior

Accuracy: 60%



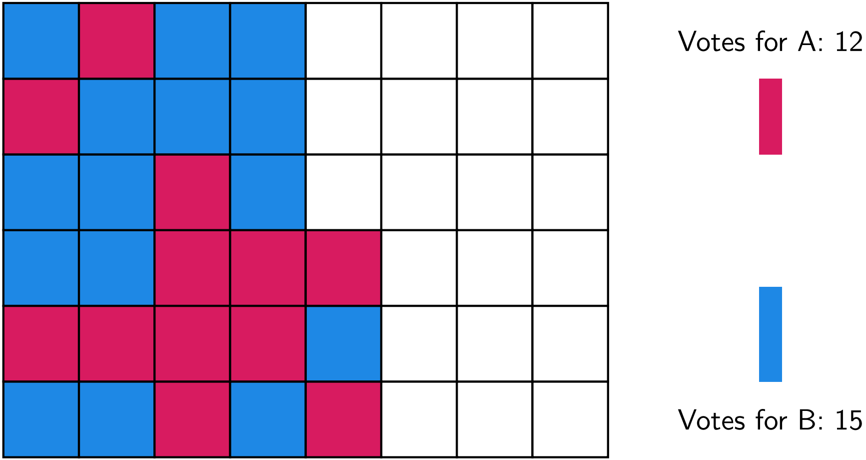
Two Candidates Election with Uniform Prior

Accuracy: 60%



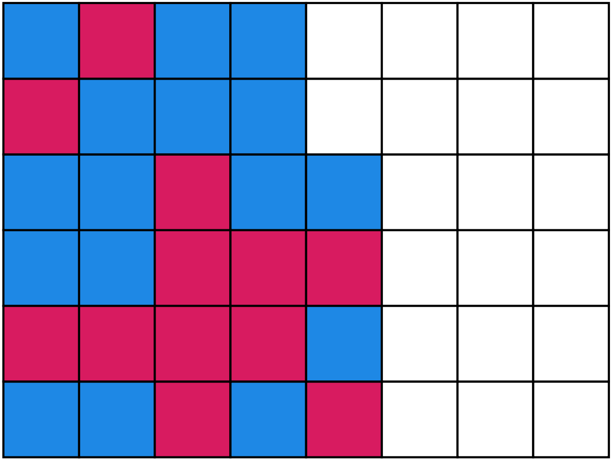
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

Accuracy: 60%

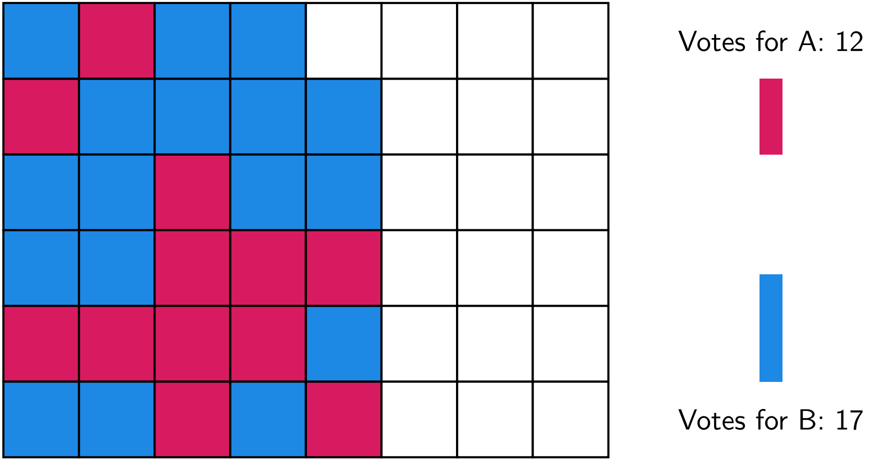


Votes for A: 12

Votes for B: 16

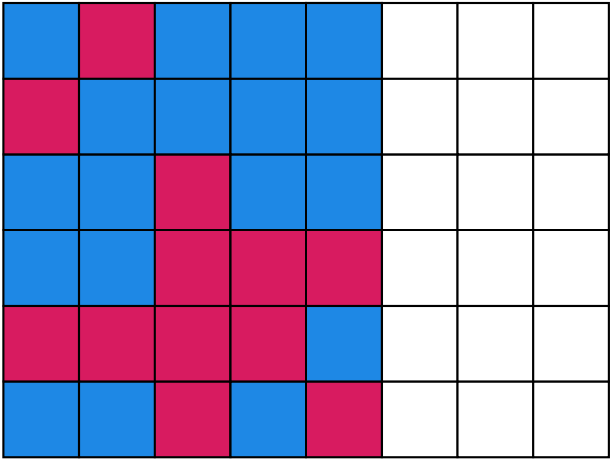
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

Accuracy: 60%

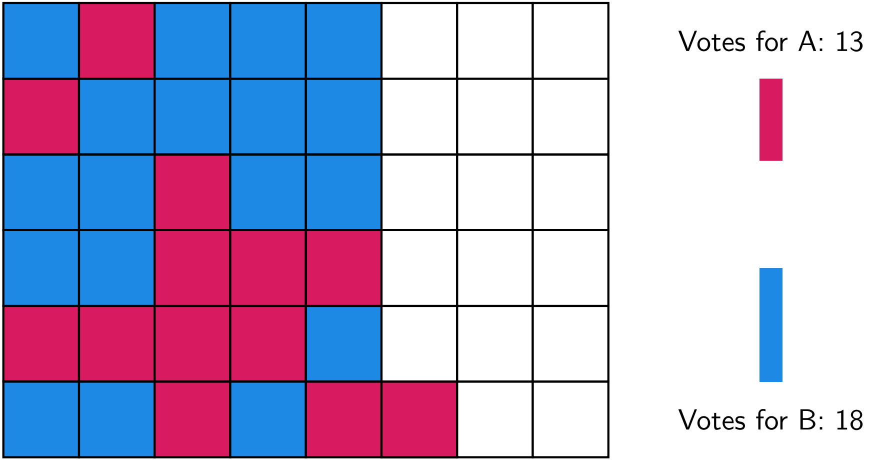


Votes for A: 12

Votes for B: 18

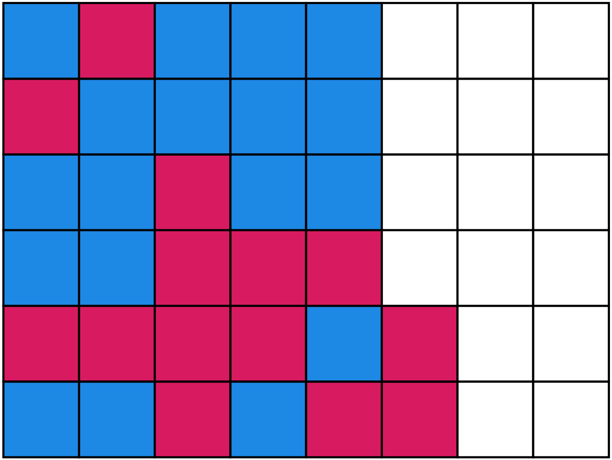
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

Accuracy: 60%

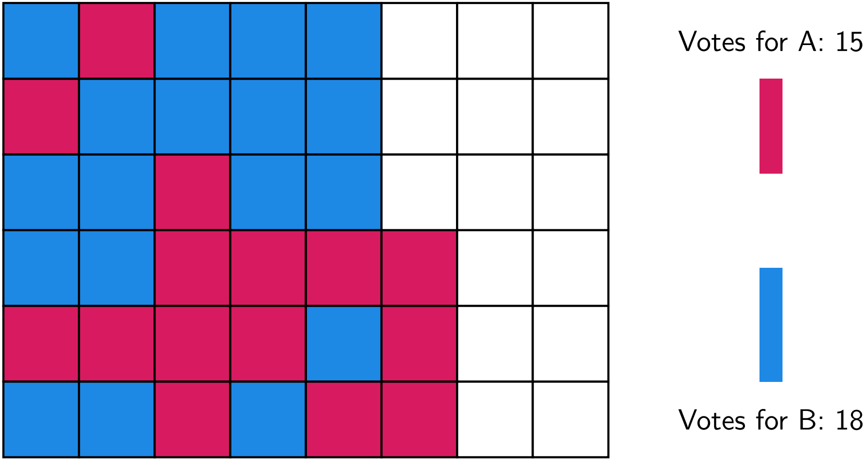


Votes for A: 14

Votes for B: 18

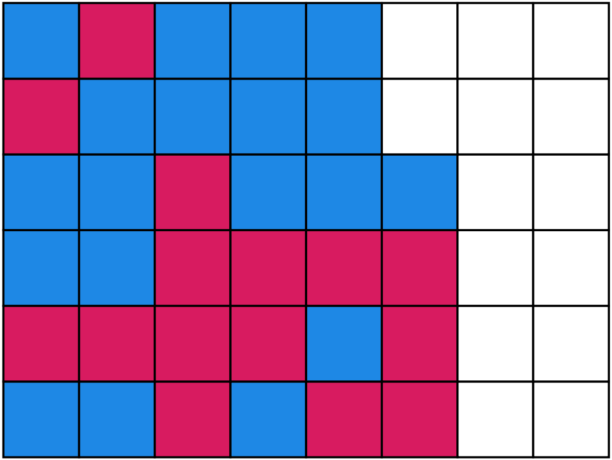
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

Accuracy: 60%

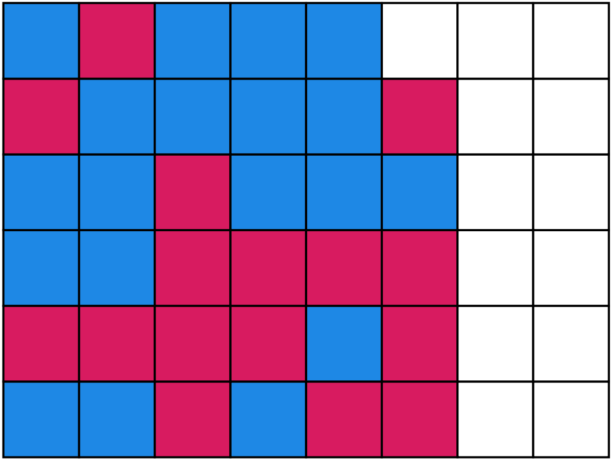


Votes for A: 15

Votes for B: 19

Two Candidates Election with Uniform Prior

Accuracy: 60%

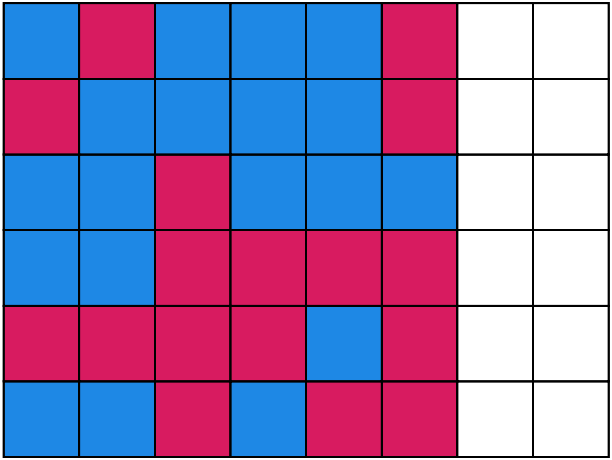


Votes for A: 16

Votes for B: 19

Two Candidates Election with Uniform Prior

Accuracy: 60%

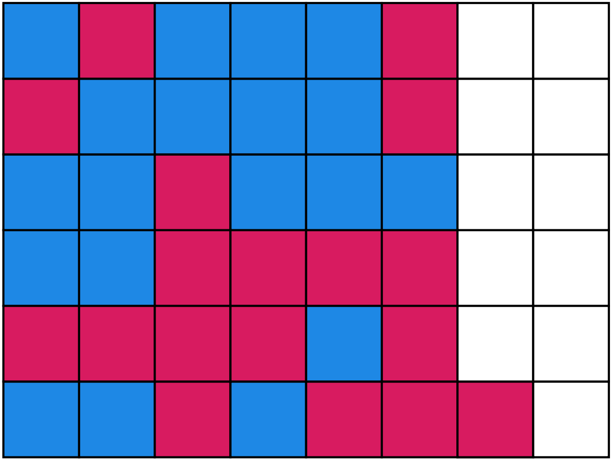


Votes for A: 17

Votes for B: 19

Two Candidates Election with Uniform Prior

Accuracy: 60%

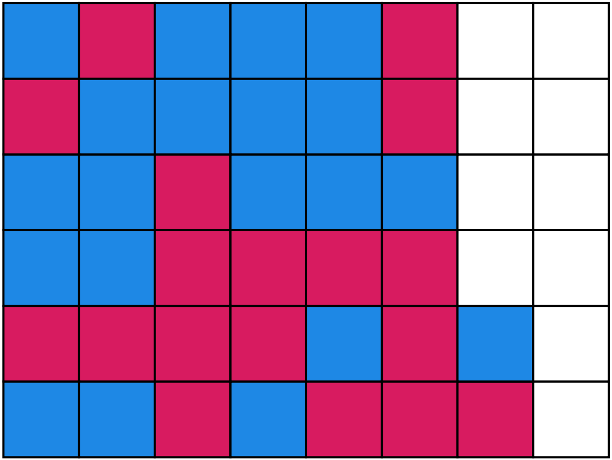


Votes for A: 18

Votes for B: 19

Two Candidates Election with Uniform Prior

Accuracy: 60%

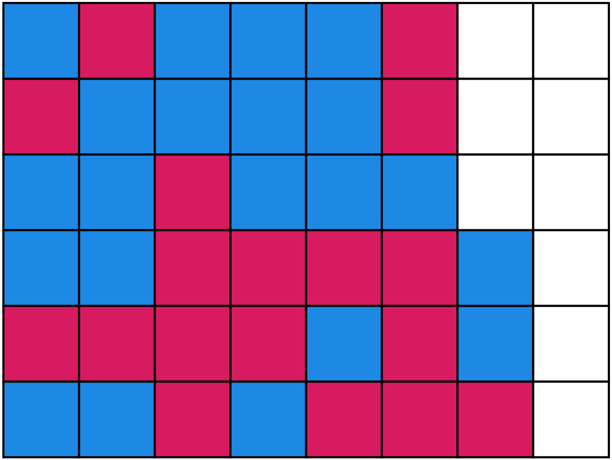


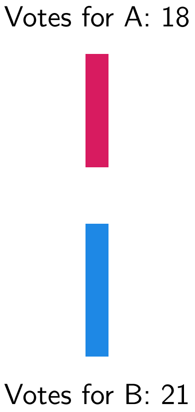
Votes for A: 18

Votes for B: 20

Two Candidates Election with Uniform Prior

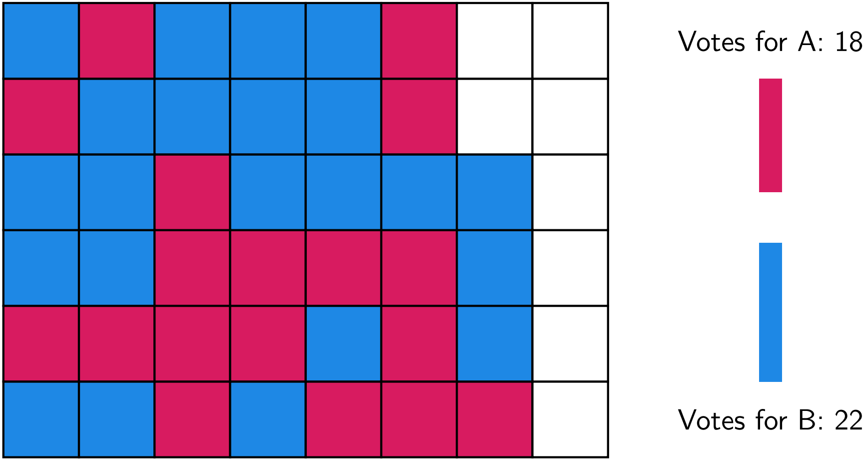
Accuracy: 60%





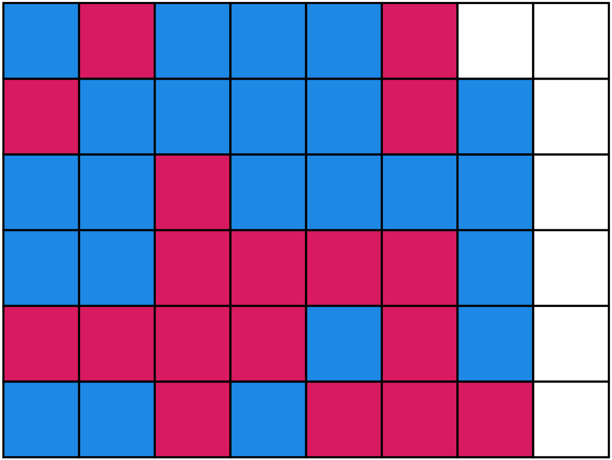
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

Accuracy: 60%

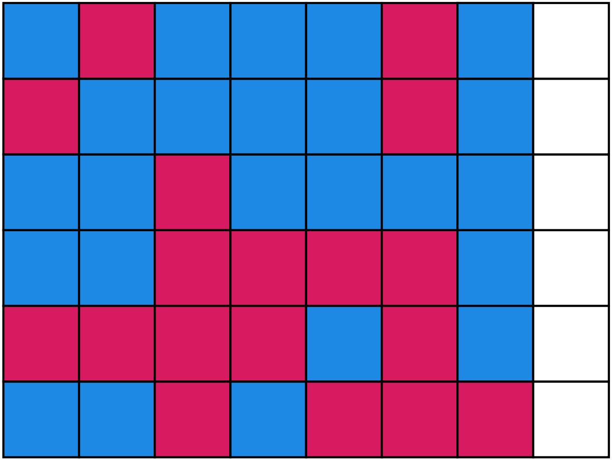


Votes for A: 18

Votes for B: 23

Two Candidates Election with Uniform Prior

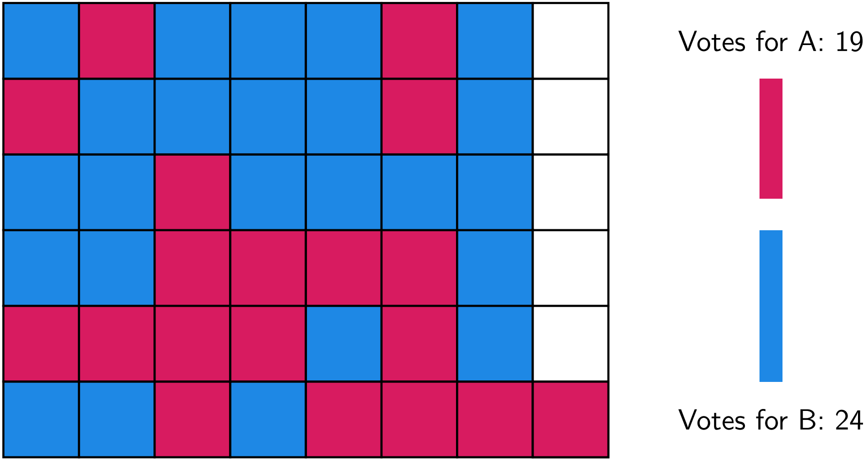
Accuracy: 60%



Votes for A: 18

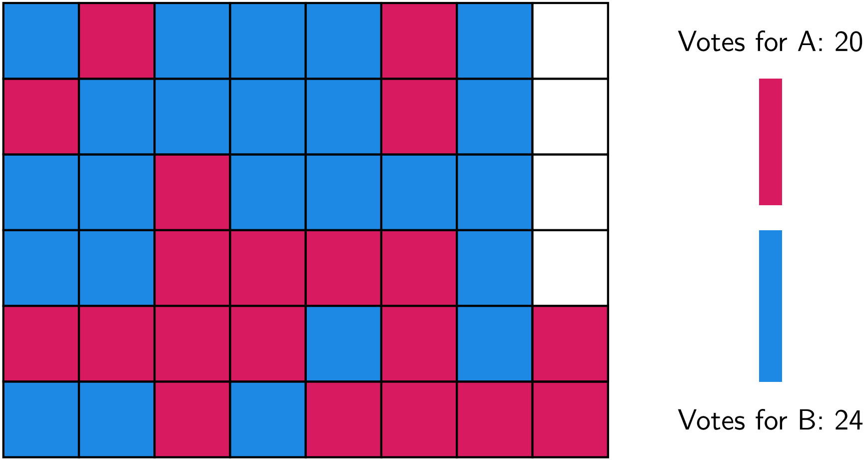
Votes for B: 24

Accuracy: 60%



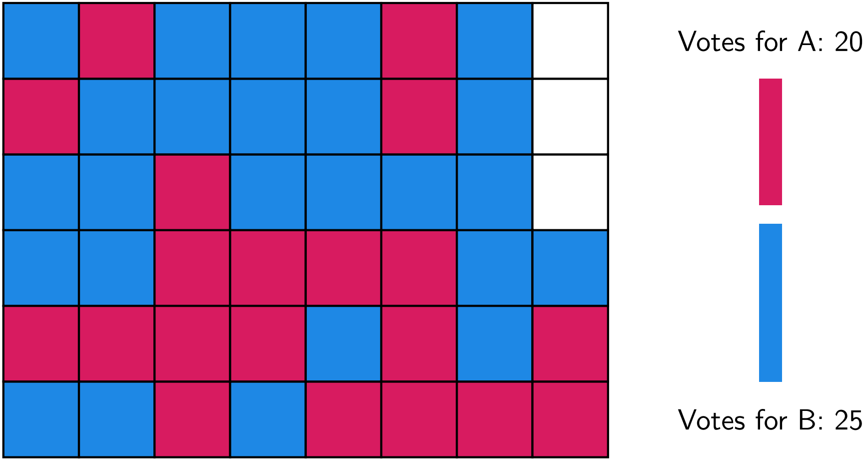
Two Candidates Election with Uniform Prior

Accuracy: 60%



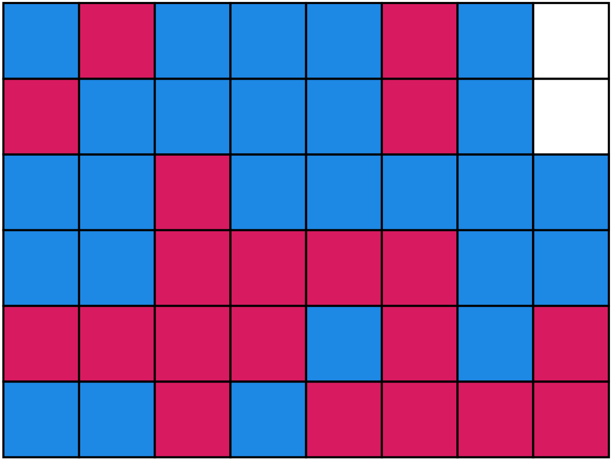
Two Candidates Election with Uniform Prior

Accuracy: 60%



Two Candidates Election with Uniform Prior

Accuracy: 60%

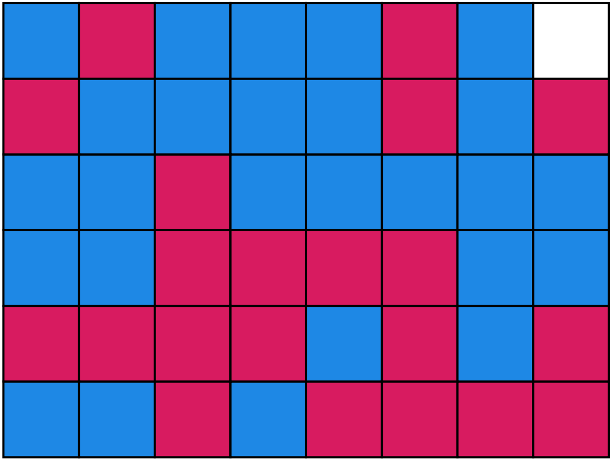


Votes for A: 20

Votes for B: 26

Two Candidates Election with Uniform Prior

Accuracy: 60%

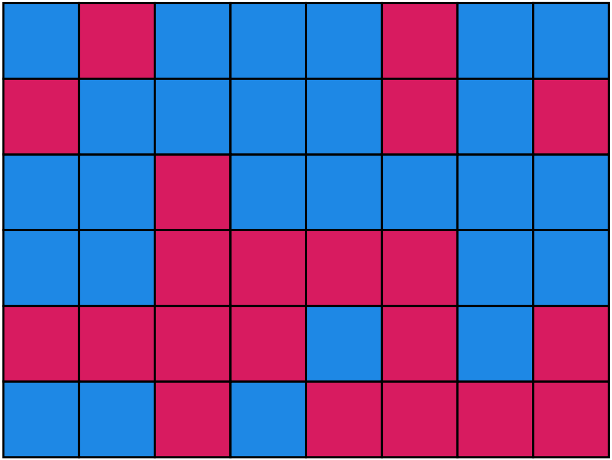


Votes for A: 21

Votes for B: 26

Two Candidates Election with Uniform Prior

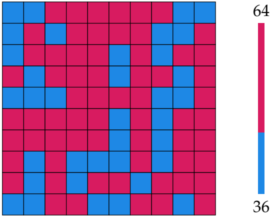
Accuracy: 60%

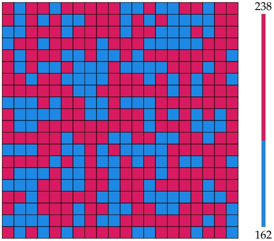


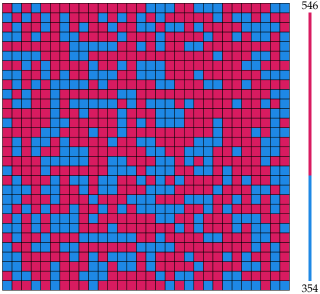
Votes for A: 21

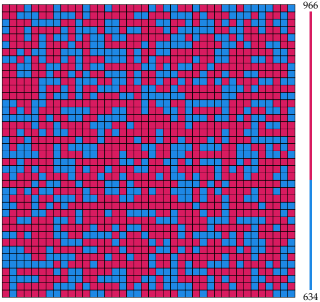
Votes for B: 27

Increasing Number of Voters with Accuracy 60%









Condorcet Jury Theorem

THEOREM:

For an election with *two candidates* and n voters, if the voters correctly identify the ground truth with probability $1/2 < p \leq 1$ and do so *independently*, then the *majority rule* selects the ground truth with probability *1* as $n \to +\infty$.

• De Condorcet "Essai sur l'Application de l'Analyse à la Probabilité des Décisions Rendues à la Pluralité des Voix" (1785)

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↳ This is the first application of the maximum likelihood approach that we know of!

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2. Maximum Likelihood Approach

Maximum Likelihood Approach

Basic Definitions and First Results

Likelihood

$$L(\boldsymbol{V}, \theta) = \prod_{V \in \boldsymbol{V}} \mathbb{P}(V \mid \theta)$$

Likelihood

Set of ballots
(Profile)

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Parameter of the model
(Ground truth)

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Parameter of the model
(Ground truth)

Probabilistic model
(Noise model)

Maximum Likelihood Estimator

$$R(\boldsymbol{V}) = \arg\max_{\theta} L(\boldsymbol{V}, \theta) = \arg\max_{\theta} \prod_{V \in \boldsymbol{V}} \mathbb{P}(V \mid \theta)$$

Maximum Likelihood Estimator

Maximum Likelihood Estimator
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Parameter
(Ground truth)

Likelihood

MLE and Classical Voting Setting

	MLE for Winner	not MLE for Winner
MLE for Ranking	Scoring rules: Borda, veto, plurality...	Weird rules
not MLE for Ranking	STV	Bucklin, Copeland, maximin, ranked pairs...

• Conitzer and Sandholm "Common voting rules as maximum likelihood estimators" (2006)
• Conitzer, Rognlie, and Xia "Preference Functions that Score Rankings and Maximum Likelihood Estimation" (2009)

Maximum Likelihood Approach

The Case of Approval Ballots

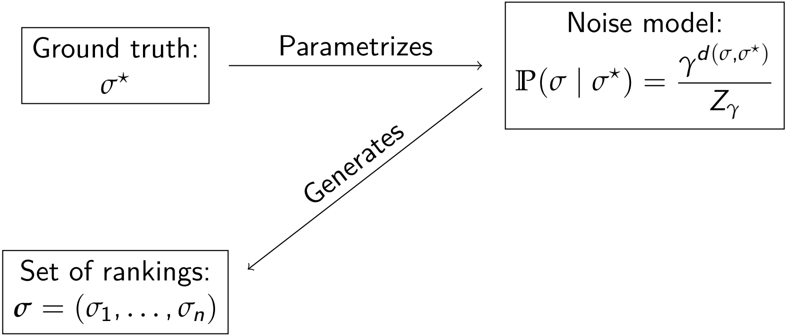
Using Approval Ballots

Ground truth:
$\sigma^\star$

Using Approval Ballots

Ground truth: $\sigma^\star$ —Parametrizes→ Noise model: $\mathbb{P}(\sigma \mid \sigma^\star) = \dfrac{\gamma^{d(\sigma,\sigma^\star)}}{Z_\gamma}$

Using Approval Ballots



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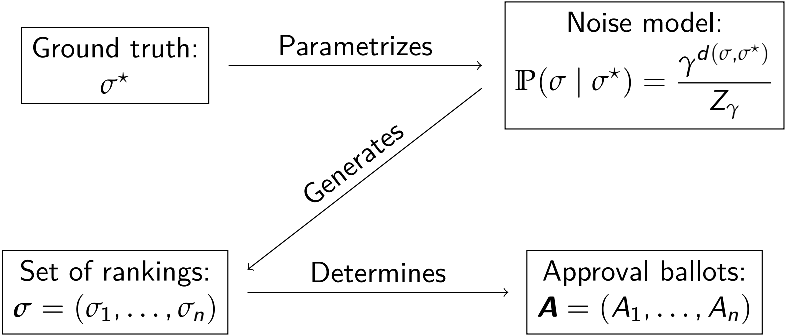
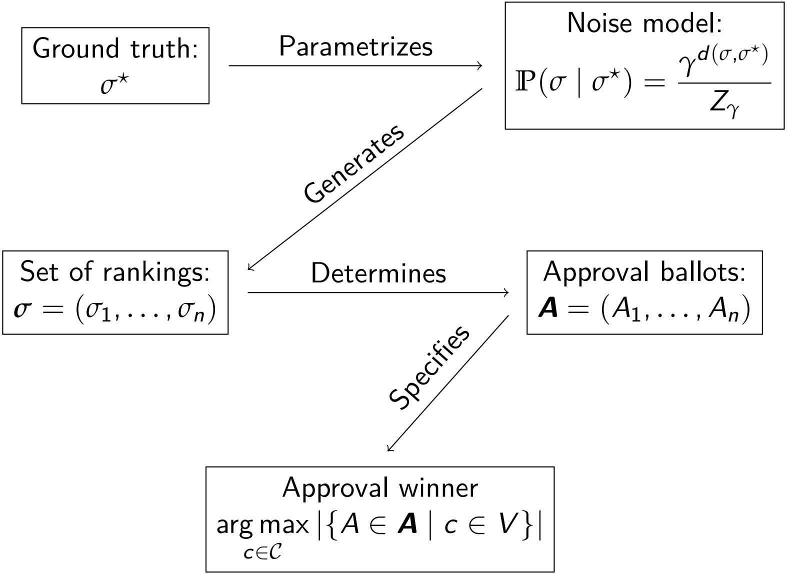
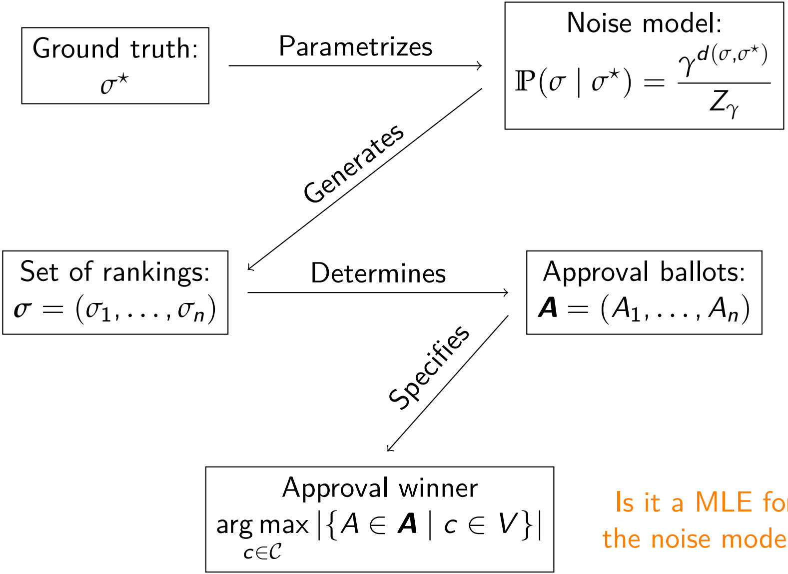
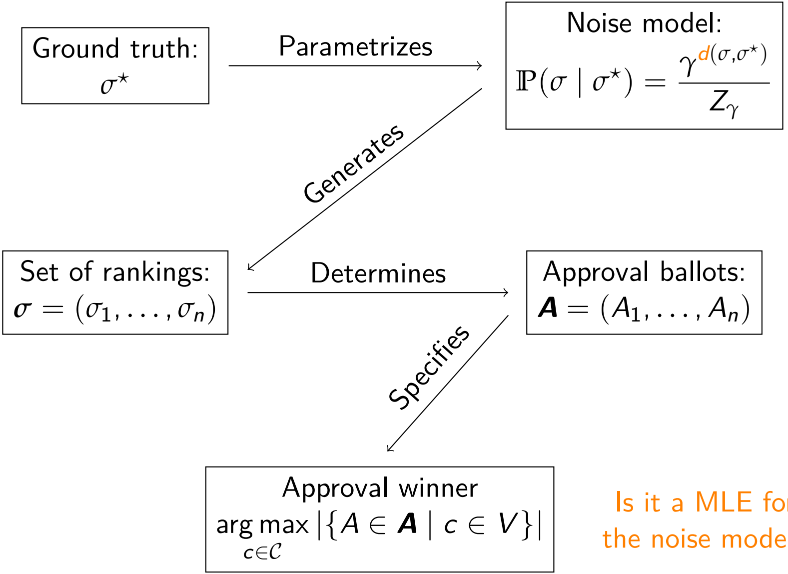



Ground truth:
$\sigma^\star$
Parametrizes
Noise model:
$\mathbb{P}(\sigma \mid \sigma^\star) = \frac{\gamma^{d(\sigma,\sigma^\star)}}{Z_\gamma}$
Generates
Set of rankings:
$\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$
Determines
Approval ballots:
$\boldsymbol{A} = (A_1, \ldots, A_n)$
Specifies
Approval winner
$\arg\max_{c \in \mathcal{C}} |\{A \in \boldsymbol{A} \mid c \in V\}|$



Is it a MLE for the noise model?



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Using Approval Ballots

THEOREM:

With the *Kendall tau* distance the set of MLE best alternatives coincides with the set of approval winners.

- Procaccia and Shah "Is Approval Voting Optimal Given Approval Votes?" (2015)

Using Approval Ballots

THEOREM:

With the *Kendall tau* distance the set of MLE best alternatives coincides with the set of approval winners.

With *plurality* or *veto* ballots, approval voting is an MLE for every "relevant" distance.

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Looking for Specific Objectives

Select k alternatives so to maximize the probability of containing:

1. the *top alternative* of the ground truth ranking,
2. the *top k alternatives* of the ground truth ranking,
3. the *top k alternatives* of the ground truth ranking *in the right order*.

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3. the *top k alternatives* of the ground truth ranking *in the right order*.

THEOREM:

All three objectives are *NP-hard* to achieve under Mallows' model.

They are *tractable* in very *noisy situation* ($\gamma \approx 1$).

• Procaccia, Reddi, and Shah "A maximum likelihood approach for selecting sets of alternatives" (2012)

3. Sample Complexity

Sample Complexity

Some Definitions

Accuracy of a Rule

$$Acc(R, k) = \left(\sum_{\boldsymbol{V} \in \mathcal{L}(A)^k} \mathbb{P}(\boldsymbol{V} \mid \sigma^\star)\mathbb{P}(R(\boldsymbol{V}) = \sigma^\star) \right)$$

Accuracy of a Rule

Accuracy of rule R with k samples

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A profile of size k

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A ranking of the alternatives

A profile of size k

Probability for R to return $\sigma^\star$ on $\boldsymbol{V}$

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$$\mathcal{SC}(R, \epsilon) = \min \{k \mid Acc(R, k) \geq 1 - \epsilon\}$$

Sample Complexity

Sample Complexity in Practice

The Kemeny Rule is Optimal for Mallows' Model

<u>THEOREM</u>:

Given $\epsilon > 0$, the *Kemeny rule* with uniform tie-breaking is such that for Mallows' mode and for every rule R, we have:

$$\mathcal{SC}(KEM, \epsilon) \leq \mathcal{SC}(R, \epsilon).$$

- Caragiannis, Procaccia, and Shah "When do Noisy Votes Reveal the Truth?" (2013)

Number of Samples Required

THEOREM:

For any $\epsilon > 0$, the *Kemeny rule* returns the ground truth with probability $1 - \epsilon$ given $\mathcal{O}(\ln(|A|/\epsilon))$ and no rule can do better.

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Number of Samples Required

THEOREM:

For any $\epsilon > 0$, the *Kemeny rule* returns the ground truth with probability $1 - \epsilon$ given $\mathcal{O}(\ln(|A|/\epsilon))$ and no rule can do better.

↳ Also holds for pairwise-majority consistent rules.

- Caragiannis, Procaccia, and Shah "When do Noisy Votes Reveal the Truth?" (2013)

Scoring Rules

- The plurality rule sometimes requires *exponentially* many samples for Mallows' model.
- Positional scoring rules with distinct weights require a *polynomial* number of samples from Mallows' model.

• Caragiannis, Procaccia, and Shah "When do Noisy Votes Reveal the Truth?" (2013)

4. Robustness to Noise

Robustness to Noise

Definitions, Again!

Accuracy in the Limit

Definition:

A rule R is *accurate in the limit* for a noise model if for every $\epsilon > 0$, there exists n_ϵ such that for every profile of size at least n_ϵ, R returns the *ground truth* with probability $1 - \epsilon$.

Monotone Robust Rules

Definition:

A noise model is *d-monotonic* if for any σ, σ', we have:

$$\mathbb{P}(\sigma \mid \sigma^\star) > \mathbb{P}(\sigma' \mid \sigma^\star) \iff d(\sigma, \sigma^\star) < d(\sigma', \sigma^\star).$$

Definition:

A rule is *monotone robust* against d if it is accurate in the limit for *every* d-monotonic noise model.

Robustness to Noise

Pairwise Majority Consistent Rules

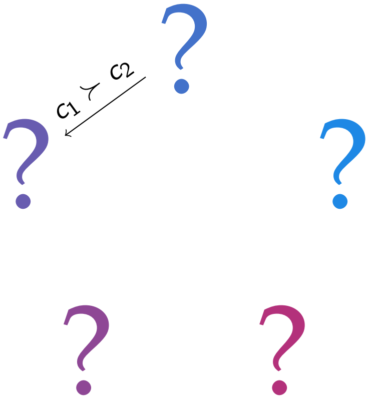
Pairwise Majority Consistency

?

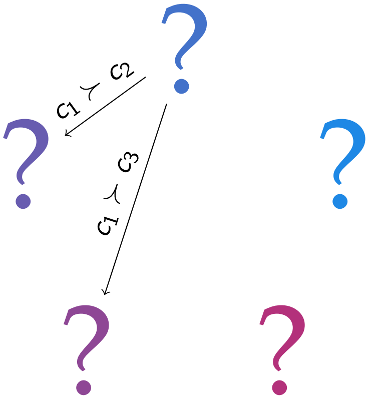
? ?

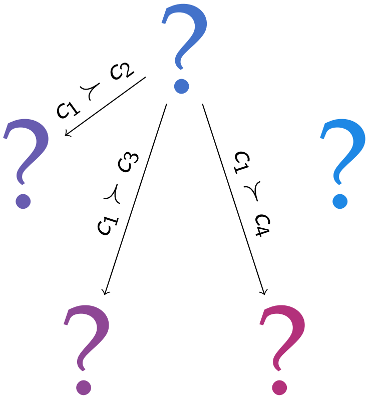
? ?

Pairwise Majority Consistency

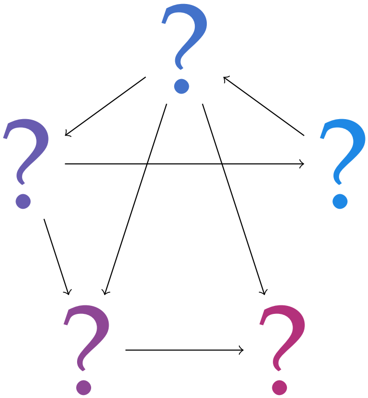


Pairwise Majority Consistency




$c_1 \succ c_2$
$c_1 \succ c_3$
$c_1 \succ c_4$

Pairwise Majority Consistency



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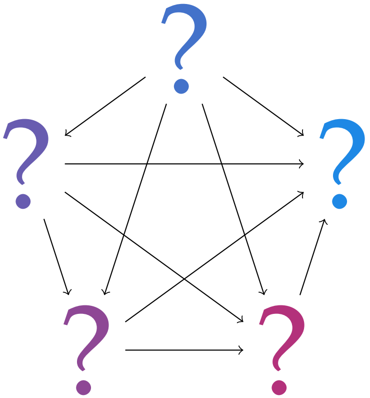

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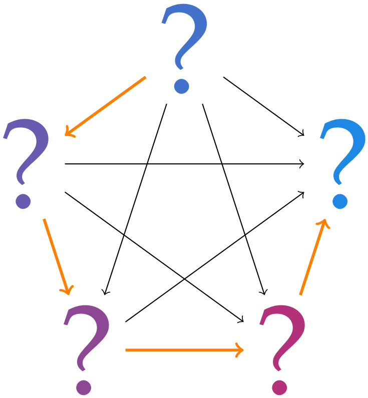

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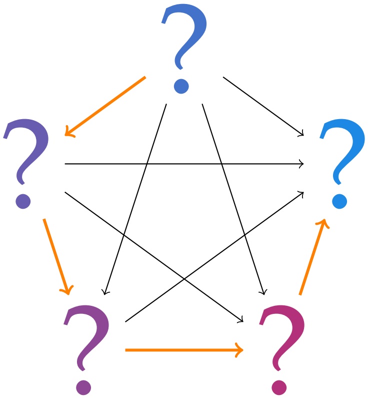
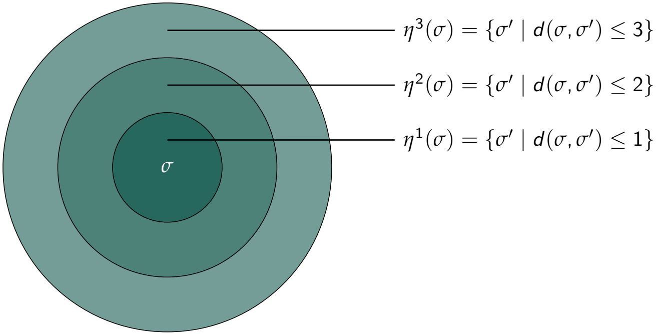


DEFINITION:

A rule R is PM-consistent if it outputs the *Condorcet order* when the PM graph is *complete* and *acyclic*.

Majority Concentric Distances



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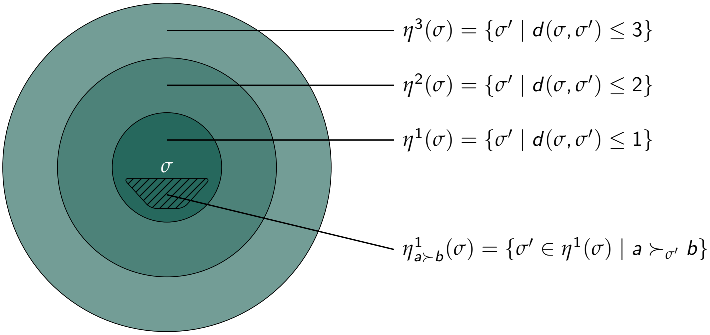

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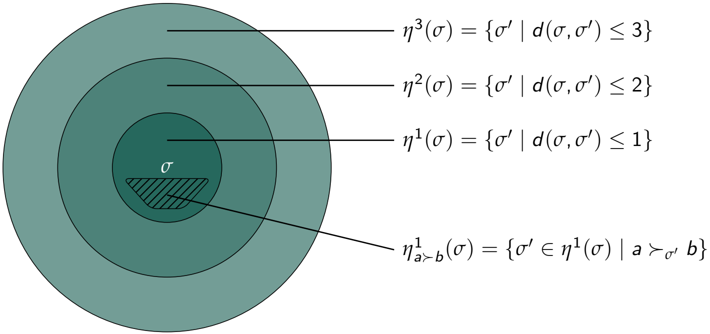

DEFINITION:

A distance d if *majority concentric* if for every σ, every a, b such that $a \succ_\sigma b$ and every k we have:

$$|\eta^k_{a \succ b}(\sigma)| \geq |\eta^k_{b \succ a}(\sigma)|$$

Monotone Robust PM Consistent Rules

<u>THEOREM</u>:

All *PM consistent rules* are monotone robust against d if and only if d is *majority concentric*.

- Caragiannis, Procaccia, and Shah "When do Noisy Votes Reveal the Truth?" (2013)

Multiwinner voting

<u>THEOREM</u>:

Multiwinner approval voting is d-monotone robust if and only if d is *majority concentric*.

- Caragiannis, Kaklamanis, Karanikolas, and Krimpas "Evaluating Approval-Based Multiwinner Voting in Terms of Robustness to Noise" (2020)

Robustness to Noise

Gloablly Robust Rules

Uniquely Robust Rules

<u>Theorem</u>:

Modal ranking is the only generalized scoring rule that is monotone robust against *all* distances.

• Caragiannis, Procaccia, and Shah "Modal Ranking: A Uniquely Robust Voting Rule." (2014)
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5. Conclusion and Future Directions

Concepts for Epistemic Social Choice

- *Maximum Likelihood Approach:* Which outcome should we select given that agents form their preferences following a specific noise model?

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Concepts for Epistemic Social Choice

- *Maximum Likelihood Approach:* Which outcome should we select given that agents form their preferences following a specific noise model?
- *Sample Complexity:* How many samples do we need to achieve a suitable accuracy?
- *Robustness to Noise:* Does the rule return the ground truth with high probability when there are infinitely many ballots? Is it true for classes of noise model based on a distance?

Missing Parts

- Generalizations of the Condorcet Jury Theorem
- Epistemic social choice literature in Economics, Political Science ....
- Other estimators, criteria, objectives, ...

• Bovens and Rabinowicz "Democratic answers to complex questions–an epistemic perspective" (2006)
• Pivato "Voting Rules as Statistical Estimators" (2013)
• Xia "Statistical Properties of Social Choice Mechanisms" (2014)
• Elkind and Slinko "Rationalizations of Voting Rules" (2016)
• Pivato "Realizing epistemic democracy" (2019)

Possible Future Directions

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- Investigating different probability models with non-uniform distributions, dependencies to other features of the models...
- Looking into the links between various complexity classes: elicitation complexity, sample complexity, communication complexity...