Almost Group Envy-free Allocation of Indivisible Goods and Chores

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Research Question

How to define fairness criteria for *groups of agents* when considering the allocation of indivisible *goods and chores*?

The main idea is to extend the *group fairness criteria* introduced by Conitzer et al. (2019) when there are only goods to the framework of fair division of *goods and chores* introduced by Aziz et al. (2019).

Setting

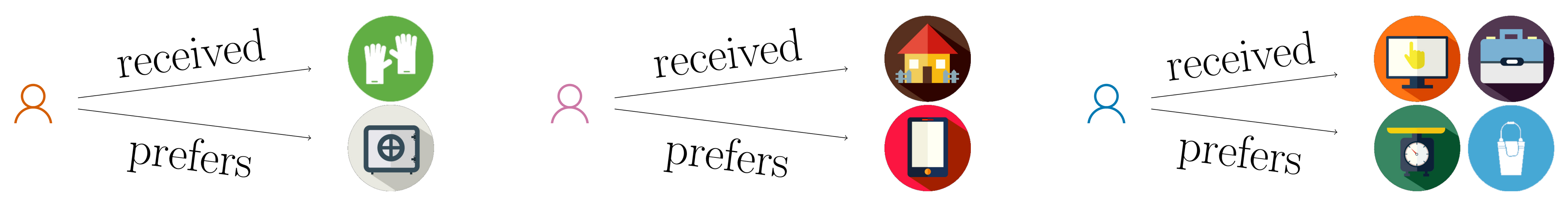
We are interested in *Fair Division* problems where:

- Items are *indivisible*
- Items can be considered as *goods* and/or *chores*
- Agents have *additive* preferences

Group envy-freeness

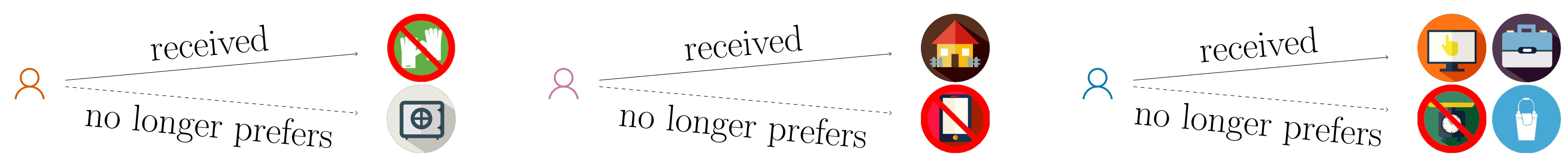
An allocation is GEF if there are *no* two groups S and T with $|S| = |T|$ such that:

meaning that there is a *reallocation* of the items received by agents in T *to agents* S that *Pareto-dominates* the current allocation for agents in S:



Group envy-freeness up to one item

We can decide *not to consider an item* for each agent in S when checking whether she prefers the reallocation. If that eliminates the envy, the allocation is GEF1:



An allocation π is GEF1 if for every two groups S and T, $|S| = |T| \neq 0$,

1. for every reallocation π' of the items in T to the agents in S,
2. for every agent $i \in S$, there exists an item o_i in what she received either in the current allocation or in the reallocation such that,
3. by removing all o_i, the reallocation π' does not Pareto-dominate π.

Theorems

When agents have *additive* and *identical* preferences, a GEF1 allocation *always exists* and can be computed in *polynomial time*.

When agents have *additive* and *ternary symmetric*[1] preferences, a GEF1 allocation *always exists* and can be computed in *polynomial time*.

[1] For every agent, the preferences of the single items are always in $\{\alpha, 0, -\alpha\}$.

Theorem

Checking whether an allocation is GEF1 is *coNP-complete* for additive preferences and where there are only goods, only chores or both goods and chores.

Aziz, H.; Caragiannis, I.; Igarashi, A.; and Walsh, T. 2019. Fair Allocation of Indivisible Goods and Chores. In *Proc. of 28th IJCAI*.

Conitzer, V.; Freeman, R.; Shah, N.; and Vaughan, J. W. 2019. Group Fairness for the Allocation of Indivisible Goods. In *Proc. of 33rd AAAI Conference*.